Illustrative Mathematics

4.NF Using Benchmarks to Compare Fractions

Alignment 1: 4.NF.A.2

Melissa gives her classmates the following explanation for why $\frac{1}{5} < \frac{11}{40}$:

> *I can compare both $\frac{1}{5}$ and $\frac{11}{40}$ to $\frac{1}{4}$.*
>
> *Since $\frac{1}{5}$ and $\frac{1}{4}$ are unit fractions and fifths are smaller than fourths, I know that $\frac{1}{5} < \frac{1}{4}$.*
>
> *I also know that $\frac{1}{4}$ is the same as $\frac{10}{40}$, so $\frac{11}{40}$ is bigger than $\frac{1}{4}$.*
>
> *Therefore $\frac{1}{5} < \frac{11}{40}$.*

a. Explain each step in Melissa's reasoning. Is she correct?

b. Use Melissa's strategy to compare $\frac{29}{60}$ and $\frac{45}{88}$, this time comparing both fractions with $\frac{1}{2}$.

c. Use Melissa's strategy to compare $\frac{8}{25}$ and $\frac{19}{45}$. Explain which fraction you chose for comparison and why.

Commentary:

This task is intended primarily for instruction purposes. The goal is to provide examples for comparing two fractions, $\frac{1}{5}$ and $\frac{11}{40}$ in this case, by finding a benchmark fraction which lies in between the two. In Melissa's example, she chooses $\frac{1}{4}$ as being larger than $\frac{1}{5}$ and smaller than $\frac{11}{40}$.

This is an important method for comparing fractions and one which requires a strong number sense and ability to make mental calculations. It is, however, a difficult ability to assess because the method is only appropriate when there is a clear benchmark fraction to be used. In part (c) of the problem, for example, students may see the denominator of 25 and think that $\frac{1}{5}$ or $\frac{2}{5}$ would be potential fractions to use for comparison. However, there are no fifths between these $\frac{8}{25}$ and $\frac{14}{39}$, and consequently students might spend a lot of time spinning their wheels trying to make one of those comparisons work. Both fractions are less than $\frac{1}{2}$, so identifying $\frac{1}{3}$ as a possibility for comparison hopefully will come from the students but may need to be suggested if they struggle.

Solution: 1

a. Melissa's reasoning is correct. For the first step $\frac{1}{5}$ represents one of five equal pieces that make up a whole. $\frac{1}{4}$ represents one of four equal pieces making up the *same* whole. Since there are fewer of the equal pieces of size $\frac{1}{4}$ making up the same whole, $\frac{1}{5} < \frac{1}{4}$.

   Next, Melissa argues that $\frac{1}{4} < \frac{11}{40}$. To compare these two fractions, she is using 40 as a common denominator. To write $\frac{1}{4}$ as a fraction with 40 in the denominator means that the denominator is multiplied by 10. Multiplying the numerator by 10 also gives

$$\frac{1}{4} = \frac{10 \times 1}{10 \times 4} = \frac{10}{40}.$$

   Now $\frac{10}{40} < \frac{11}{40}$ because the denominators of these two fractions are the same and 11 equal pieces of size $\frac{1}{40}$ is more than 10 equal pieces of size $\frac{1}{40}$. So this shows that $\frac{1}{4} < \frac{11}{40}$

   Combining the work from the first two paragraphs gives

$$\frac{1}{5} < \frac{1}{4} < \frac{11}{40}$$

   and so $\frac{1}{5} < \frac{11}{40}$. Melissa's reasoning is involved but correct.

b. Using Melissa's strategy, the goal is to compare $\frac{29}{60}$ to $\frac{1}{2}$ and then to compare $\frac{45}{88}$ to $\frac{1}{2}$. For $\frac{29}{60}$ and $\frac{1}{2}$ we can compare these fractions by finding a common denominator. Since 2 is a factor of 60 we can use 60 as a common denominator. To write $\frac{1}{2}$ with a denominator of 60 we need to multiply the denominator (and numerator) by 30:

$$\frac{1}{2} = \frac{30 \times 1}{30 \times 2} = \frac{30}{60}.$$

   Now we can see that $\frac{29}{60} < \frac{30}{60}$ since we are comparing 29 pieces to 30 pieces where these pieces all have the same size. So we find

$$\frac{29}{60} < \frac{1}{2}.$$

   Next, to compare $\frac{1}{2}$ to $\frac{45}{88}$ we can write $\frac{1}{2}$ with a denominator of 88, multiplying numerator and denominator by 44 this time:

$$\frac{1}{2} = \frac{44 \times 1}{44 \times 2} = \frac{44}{88}.$$

We know that $\frac{44}{88} < \frac{45}{88}$ because 44 pieces is less than 45 pieces and the pieces all have the same size. So we see that

$$\frac{1}{2} < \frac{45}{88}.$$

Combining the reasoning of the two paragraphs above gives

$$\frac{29}{60} < \frac{1}{2} < \frac{45}{88}$$

and so $\frac{45}{88}$ is greater than $\frac{29}{60}$.

c. The reasoning here will be like that of parts (a) and (b) if we can identify the benchmark fraction to compare with $\frac{8}{25}$ and $\frac{19}{45}$. Since $8 \times 3 = 24$, we have

$$\frac{1}{3} = \frac{8 \times 1}{8 \times 3} = \frac{8}{24}.$$

This is close to $\frac{8}{25}$ and this was what motivated the choice of $\frac{1}{3}$ (we will see below that $\frac{19}{45}$ is also close to $\frac{1}{3}$, making $\frac{1}{3}$ an appropriate fraction for comparison). To see which is larger, $\frac{1}{3}$ or $\frac{8}{25}$, note that $\frac{1}{25} < \frac{1}{24}$ because if a whole is broken into 24 equal sized pieces these pieces will be larger than if the same whole is broken into 25 equal sized pieces. So we can conclude that $\frac{8}{25} < \frac{8}{24}$ giving

$$\frac{8}{25} < \frac{1}{3}.$$

Since we used $\frac{1}{3}$ for comparison with $\frac{8}{25}$ we should also use $\frac{1}{3}$ for comparison with $\frac{19}{45}$. Since $45 = 15 \times 3$, we can convert the fraction $\frac{1}{3}$ to forty-fifths:

$$\frac{1}{3} = \frac{15 \times 1}{15 \times 3} = \frac{15}{45}.$$

Now $\frac{15}{45} < \frac{19}{45}$ because 15 is less than 19 and both fractions have a denominator of 45. So we have found that

$$\frac{1}{3} < \frac{19}{45}.$$

Combining the work of the previous two paragraphs we see that

$$\frac{8}{25} < \frac{1}{3} < \frac{19}{45}.$$

The key to using this method for comparing fractions is identifying a benchmark fraction for comparison. This requires either a good number sense or a lot of experience.

Another good choice for a benchmark comparison is the fraction $\frac{2}{5}$.

Since $25 = 5 \times 5$, we can convert the fraction $\frac{2}{5}$ to twenty-fifths:

$$\frac{2}{5} = \frac{5 \times 2}{5 \times 5} = \frac{10}{25}.$$

Now $\frac{8}{25} < \frac{10}{25}$ because 8 is less than 10 and both fractions have a denominator of 25. So we have found that

$$\frac{8}{25} < \frac{2}{5}.$$

Since we used $\frac{2}{5}$ for comparison with $\frac{8}{25}$, we should also use $\frac{2}{5}$ for comparison with $\frac{19}{45}$. Since $45 = 9 \times 5$, we can convert the fraction $\frac{2}{5}$ to forty-fifths:

$$\frac{2}{5} = \frac{9 \times 2}{9 \times 5} = \frac{18}{45}.$$

Now $\frac{18}{45} < \frac{19}{45}$ because 18 is less than 19 and both fractions have a denominator of 45. So we have found that

$$\frac{2}{5} < \frac{19}{45}.$$

Combining the previous work, we see that

$$\frac{8}{25} < \frac{2}{5} < \frac{19}{45}.$$