Introduction

The Bridges in Mathematics 1st edition Grade 5 CCSS Supplement is a collection of activities designed to help Bridges 1st edition students meet the Common Core State Standards for Mathematics published in 2010. These activities were drawn from Bridges Grade 5 Supplement Sets A4, A6, A9, A10, A11, A12, B1, C1, & D2. The full versions of these sets are available as free downloadable PDFs on The Math Learning Center website (bridges1.mathlearningcenter.org/b5sup).

Note: This document is set up to print double-sided (back-to-back).

Here's what you'll find in this packet:

1. CCSS Summary – page 4
   A summary of the supplement and the Common Core State Standards they address, covering the entire school year.

2. Recommended Timing for Activities & Independent Worksheets – pages 5–7
   A chart outlining the teaching order of the activities and worksheets found in this supplement.

3. Unit Planners – pages 9–24
   Planners designed to replace the Planning Guides found at the beginning of each unit in the Bridges Teachers Guides. These planners identify sessions that were omitted to make room for the supplement sets and describe the timing and direction for inserting supplement Activities and Independent Worksheets, including Homework. We suggest you insert these planners into your Bridges Teacher Guides so you can see at a glance when to teach the supplement activities throughout the school year.

   A complete list of materials required to teach the activities in each supplement set. Materials include those contained in the Bridges kits and common materials found in the classroom or at home.

5. Activities and Independent Worksheets – pages 27-446
   Activities and worksheets organized around a mathematical domain and cluster (e.g., domain: Measurement & Data; cluster: Area). Many of the activities will take an hour of instructional time, though some are shorter, requiring 30–45 minutes. Almost all the activities are hands-on and require various math manipulatives or common classroom supplies. The blacklines needed to make display masters, game materials, and student sheets are included after each activity. Some of the supplement sets in this collection include independent worksheets, designed to be completed by students in class or assigned as homework after related activities.
   Note: The activities and worksheets are presented in teaching order by unit. Sets may not follow standard page order. See timing chart for reference.

   The Common Core State Standards correlations to Bridges in Mathematics 1st edition Grade 5 include correlations to Bridges sessions, Supplement Activities & Independent Worksheets, Practice Book pages, and informal and formal assessment suggestions found in the Bridges materials and on the Bridges 1st edition support web pages.
Bridges in Mathematics & the Common Core State Standards (CCSS) Summary – Grade 5

<table>
<thead>
<tr>
<th>OPERATIONS &amp; ALG THINKING</th>
<th>NUMBER &amp; OPS BASE 10</th>
<th>FRACTIONS</th>
<th>MEASUREMENT &amp; DATA</th>
<th>GEOMETRY</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Numerical Expressions</td>
<td>• Operations with Multi-Digit Whole Numbers &amp; Decimals</td>
<td>• Add, Subtract, Multiply, and Divide Fractions</td>
<td>• Measurement Conversions, Data, Volume</td>
<td>• Coordinate Grids, Classify Two-Dimensional Shapes</td>
</tr>
<tr>
<td>• Patterns &amp; Relationships</td>
<td>Bridges Units: 1, 2, 4, 6</td>
<td>Bridges Units: 4, 6</td>
<td>Bridges Units: 3</td>
<td>Bridges Units: 3</td>
</tr>
<tr>
<td>Bridges Units: 1, 2, 7</td>
<td>Number Corner: Oct–Feb, Mar, Apr</td>
<td>Number Corner: Nov, Mar, Apr</td>
<td>Number Corner: Jan, Apr</td>
<td>Number Corner: Sep, Oct, Mar</td>
</tr>
<tr>
<td>Number Corner: Sep</td>
<td>Supplemental Sets: A4, A11</td>
<td>Supplemental Sets: A6, A9, A11, A12</td>
<td>Supplemental Set: D2</td>
<td>Supplemental Set: C1</td>
</tr>
<tr>
<td>Supplemental Sets: A10, B1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Pacing Guide (163 sessions total; school calendars determine specific timing)

<table>
<thead>
<tr>
<th>SEP 18 SESSIONS</th>
<th>OCT to MID-NOV 21 SESSIONS</th>
<th>MID-NOV to DEC 26 SESSIONS</th>
<th>JAN 23 SESSIONS</th>
<th>FEB to MID-MAR 27 SESSIONS</th>
<th>MID-MAR to MID-APR 15 SESSIONS</th>
<th>MID-APR to MID-MAY 21 SESSIONS</th>
<th>MID-MAY to JUNE 12 SESSIONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Units</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>UNIT 1 18 Sessions</td>
<td>Connecting Math Topics</td>
<td>UNIT 2 21 Sessions</td>
<td>Multi-Digit Multiplication &amp; Division</td>
<td>UNIT 3 16 Sessions</td>
<td>Geometry &amp; Measurement</td>
<td>UNIT 4 23 Sessions</td>
<td>Multiplication, Division &amp; Fractions</td>
</tr>
<tr>
<td>SET C1: Triangles &amp; Quadrilaterals</td>
<td>4 Sessions, 6 IWS</td>
<td>SET D2: Volume</td>
<td>6 Sessions, 5 IWS</td>
<td>SET A6: Fractions 2 Sessions, 3 IWS</td>
<td>SET A9: Multiplying Fractions 7 Sessions, 5 IWS</td>
<td>SET A4: Long Division</td>
<td>2 Sessions</td>
</tr>
<tr>
<td>IWS = Independent Worksheets for homework or seatwork</td>
<td></td>
<td></td>
<td></td>
<td>SET A11: Multiplying &amp; Dividing Decimals 8 Sessions, 8 IWS</td>
<td>SET A12: Dividing Fractions &amp; Whole Numbers 8 Sessions, 3 IWS</td>
<td>SET A10: Introducing Integers</td>
<td>3 Sessions, 3 IWS</td>
</tr>
<tr>
<td>SET B1: Diagrams &amp; Equations</td>
<td>1 Sessions, 2 IWS</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

CCSS Supplement Sets

<table>
<thead>
<tr>
<th>SEP</th>
<th>OCT</th>
<th>NOV</th>
<th>DEC</th>
<th>JAN</th>
<th>FEB</th>
<th>MAR</th>
<th>APR</th>
<th>MAY/JUN</th>
</tr>
</thead>
<tbody>
<tr>
<td>Basic facts; repeating patterns; 2-D shapes; line and rotational symmetry; probability and data</td>
<td>Basic facts (×); factors; repeating patterns; similar figures; area; range; mode; median and mean</td>
<td>Multi-digit multiplication; fractions and decimals; number patterns; capacity</td>
<td>Rounding and estimation; multi-digit multiplication; repeating patterns; congruent figures; line and rotational symmetry; weight; and line graphs</td>
<td>Multi-digit multiplication; number patterns; temperature; line graphs; volume; and data analysis</td>
<td>Multi-digit division; fractions and decimals; patterns and functions; probability and data</td>
<td>Primes; composites; factors and multiples; fractions and decimals; coordinate grids; elapsed time; and line graphs</td>
<td>Place value; fractions; number patterns; volume and surface area</td>
<td>Multi-digit division; number patterns; angle measure and transformations</td>
</tr>
</tbody>
</table>
# Recommended Timing for Activities & Independent Worksheets

Activities listed in recommended teaching order.

<table>
<thead>
<tr>
<th>UNIT 1</th>
<th>Set, Strand &amp; Topic</th>
<th>Activity or Independent Worksheet</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>No supplement sets used</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>UNIT 2</th>
<th>Set, Strand &amp; Topic</th>
<th>Activity or Independent Worksheet</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>No supplement sets used</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>UNIT 3</th>
<th>Set, Strand &amp; Topic</th>
<th>Activity or Independent Worksheet</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Set C1: Geometry: Triangles &amp; Quadrilaterals</td>
<td>Activity 1: Classifying Triangles</td>
<td>C1.1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Ind. Worksheet 1: More Geoboard Triangles</td>
<td>C1.43</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Activity 2: Sorting &amp; Classifying Quadrilaterals</td>
<td>C1.13</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Ind. Worksheet 2: Color &amp; Construct Triangles</td>
<td>C1.45</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Activity 3: Finding the Perimeter &amp; Area of a Parallelogram</td>
<td>C1.25</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Ind. Worksheet 3: Classifying Quadrilaterals</td>
<td>C1.47</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Activity 4: Three Mathematical Ideas</td>
<td>C1.35</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Ind. Worksheet 4: Quad Construction</td>
<td>C1.51</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Ind. Worksheet 5: Perimeter &amp; Area Puzzles</td>
<td>C1.53</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Ind. Worksheet 6: Ebony’s Quilt</td>
<td>C1.55</td>
<td></td>
</tr>
<tr>
<td>Set D2: Measurement: Volume</td>
<td>Activity 1: Introducing Volume</td>
<td>D2.1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Activity 2: More Paper Boxes</td>
<td>D2.7</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Ind. Worksheet 1: Volume Review</td>
<td>D2.35</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Ind. Worksheet 2: The Camping Trip</td>
<td>D2.39</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Activity 3: Fish Tank Volume</td>
<td>D2.13</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Ind. Worksheet 3: Tank Volume</td>
<td>D2.41</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Activity 4: Number of Fish</td>
<td>D2.19</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Ind. Worksheet 4: Third Tank Needs Fish</td>
<td>D2.43</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Activity 5: Fish Tank Plants with a Line Plot</td>
<td>D2.25</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Ind. Worksheet 5: Water for the Fish Tank</td>
<td>D2.45</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Activity 6: Water for the Fish Tank Assessment</td>
<td>D2.33</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>UNIT 4</th>
<th>Set, Strand &amp; Topic</th>
<th>Activity or Independent Worksheet</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>No supplement sets used</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
### UNIT 6

<table>
<thead>
<tr>
<th>Set, Strand &amp; Topic</th>
<th>Activity or Independent Worksheet</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Set A6: Number &amp; Operations: Fraction Concepts</strong></td>
<td>Activity 1: Simplify &amp; Compare</td>
<td>A6.1</td>
</tr>
<tr>
<td></td>
<td>Ind. Worksheet 1: Using the Greatest Common Factor to Simplify Fractions</td>
<td>A6.19</td>
</tr>
<tr>
<td></td>
<td>Activity 2: Same-Sized Pieces</td>
<td>A6.9</td>
</tr>
<tr>
<td></td>
<td>Ind. Worksheet 2: Finding the Least Common Denominator</td>
<td>A6.21</td>
</tr>
<tr>
<td></td>
<td>Ind. Worksheet 3: LCM &amp; GCF</td>
<td>A6.23</td>
</tr>
<tr>
<td><strong>Set A11: Number &amp; Operations: Multiplying &amp; Dividing Decimals</strong></td>
<td>Activity 1: Multiplying by Powers of Ten</td>
<td>A11.1</td>
</tr>
<tr>
<td></td>
<td>Ind. Worksheet 1: Thinking about Tenths, Hundredths &amp; Thousandths</td>
<td>A11.59</td>
</tr>
<tr>
<td></td>
<td>Activity 2: Dividing by Powers of Ten</td>
<td>A11.7</td>
</tr>
<tr>
<td></td>
<td>Ind. Worksheet 2: Very Large &amp; Very Small Numbers</td>
<td>A11.61</td>
</tr>
<tr>
<td></td>
<td>Activity 3: Using Decimals to Calculate Sale Prices</td>
<td>A11.15</td>
</tr>
<tr>
<td></td>
<td>Ind. Worksheet 3: Multiplying &amp; Dividing by Powers of Ten</td>
<td>A11.63</td>
</tr>
<tr>
<td></td>
<td>Activity 4: Multiplying Decimals</td>
<td>A11.21</td>
</tr>
<tr>
<td></td>
<td>Ind. Worksheet 4: Using Landmark Fractions &amp; Percents to Multiply by Decimals</td>
<td>A11.65</td>
</tr>
<tr>
<td></td>
<td>Activity 5: Building a Deck, Using Partial Products &amp; Arrays for Decimal Multiplication</td>
<td>A11.29</td>
</tr>
<tr>
<td></td>
<td>Activity 6: Multiplying Decimals, More/Less</td>
<td>A11.35</td>
</tr>
<tr>
<td></td>
<td>Ind. Worksheet 5: Multiplying Two Decimals Numbers</td>
<td>A11.67</td>
</tr>
<tr>
<td></td>
<td>Activity 7: Dividing Decimals with Money &amp; Menus</td>
<td>A11.45</td>
</tr>
<tr>
<td></td>
<td>Ind. Worksheet 6: Comparing &amp; Multiplying Fractions &amp; Decimals</td>
<td>A11.69</td>
</tr>
<tr>
<td></td>
<td>Activity 8: Using Models &amp; Strategies to Divide with Decimals</td>
<td>A11.53</td>
</tr>
<tr>
<td></td>
<td>Ind. Worksheet 7: Olympic Swimmers</td>
<td>A11.71</td>
</tr>
<tr>
<td></td>
<td>Ind. Worksheet 8: Olympic Track Star</td>
<td>A11.73</td>
</tr>
</tbody>
</table>

### UNIT 5

<table>
<thead>
<tr>
<th>Set, Strand &amp; Topic</th>
<th>Activity or Independent Worksheet</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Set A12: Number &amp; Operations: Dividing Fractions &amp; Whole Numbers</strong></td>
<td>Activity 1: Dividing Fractions &amp; Whole Numbers Pre-Assessment</td>
<td>A12.1</td>
</tr>
<tr>
<td><strong>Set A9: Number &amp; Operations: Multiplying Fractions</strong></td>
<td>Activity 1: Geoboard Perimeters</td>
<td>A9.1</td>
</tr>
<tr>
<td></td>
<td>Activity 2: Fraction Multiplication Story Problems</td>
<td>A9.9</td>
</tr>
<tr>
<td></td>
<td>Activity 3: Using the Area Model for Multiplying Fractions</td>
<td>A9.17</td>
</tr>
<tr>
<td></td>
<td>Activity 4: Generalizations About Multiplying Fractions</td>
<td>A9.23</td>
</tr>
<tr>
<td></td>
<td>Ind. Worksheet 1: Sharing &amp; Grouping, Multiplying &amp; Dividing</td>
<td>A9.49</td>
</tr>
<tr>
<td></td>
<td>Activity 5: Target 1: Fractions</td>
<td>A9.31</td>
</tr>
<tr>
<td></td>
<td>Ind. Worksheet 2: More Fraction Multiplication</td>
<td>A9.51</td>
</tr>
<tr>
<td></td>
<td>Activity 6: Multiplying Domino Fractions</td>
<td>A9.37</td>
</tr>
<tr>
<td></td>
<td>Ind. Worksheet 3: Fraction Stories</td>
<td>A9.53</td>
</tr>
<tr>
<td></td>
<td>Activity 7: Area Word Problems with Mixed Numbers</td>
<td>A9.45</td>
</tr>
<tr>
<td></td>
<td>Ind. Worksheet 4: Using Strategies to Multiply Fractions with Mixed Numbers</td>
<td>A9.55</td>
</tr>
<tr>
<td><strong>Set A12: Number &amp; Operations: Dividing Fractions &amp; Whole Numbers</strong></td>
<td>Activity 2: Reviewing the Sharing &amp; Grouping Interpretations of Division</td>
<td>A12.7</td>
</tr>
<tr>
<td><strong>Set A9: Number &amp; Operations: Multiplying Fractions</strong></td>
<td>Ind. Worksheet 5: Domino Multiplication</td>
<td>A9.57</td>
</tr>
</tbody>
</table>
## Recommended Timing for Activities & Independent Worksheets (cont.)

### UNIT 5

<table>
<thead>
<tr>
<th>Set, Strand &amp; Topic</th>
<th>Activity or Independent Worksheet</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Set A12</strong>: Number &amp; Operations: Dividing Fractions &amp; Whole Numbers</td>
<td>Activity 3: Grouping Stories</td>
<td>A12.17</td>
</tr>
<tr>
<td></td>
<td>Ind. Worksheet 1: Sharing &amp; Grouping, Multiplying &amp; Dividing</td>
<td>A12.75</td>
</tr>
<tr>
<td></td>
<td>Activity 4: Dividing a Whole Number by a Fraction</td>
<td>A12.27</td>
</tr>
<tr>
<td></td>
<td>Ind. Worksheet 2: Operating with Fractions &amp; Whole Numbers</td>
<td>A12.77</td>
</tr>
<tr>
<td></td>
<td>Activity 5: Sharing Stories</td>
<td>A12.37</td>
</tr>
<tr>
<td></td>
<td>Activity 6: Dividing a Fraction by a Whole Number</td>
<td>A12.49</td>
</tr>
<tr>
<td></td>
<td>Ind. Worksheet 3: More Fractions &amp; Whole Numbers</td>
<td>A12.79</td>
</tr>
<tr>
<td></td>
<td>Activity 7: The Division Poster Project</td>
<td>A12.59</td>
</tr>
<tr>
<td></td>
<td>Activity 8: Dividing Fractions &amp; Whole Numbers Post-Assessment</td>
<td>A12.65</td>
</tr>
</tbody>
</table>

### UNIT 7

<table>
<thead>
<tr>
<th>Set, Strand &amp; Topic</th>
<th>Activity or Independent Worksheet</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Set A10</strong>: Number &amp; Operations: Integers</td>
<td>Activity 1: Introducing Integers</td>
<td>A10.1</td>
</tr>
<tr>
<td></td>
<td>Ind. Worksheet 1: Negative &amp; Positive Temperature</td>
<td>A10.29</td>
</tr>
<tr>
<td></td>
<td>Activity 2: Integer Tug O’ War</td>
<td>A10.13</td>
</tr>
<tr>
<td></td>
<td>Ind. Worksheet 2: Temperature &amp; Elevation Riddles</td>
<td>A10.31</td>
</tr>
<tr>
<td></td>
<td>Activity 3: 4-Quadrant Battleship</td>
<td>A10.23</td>
</tr>
<tr>
<td></td>
<td>Ind. Worksheet 3: Shapes on a 4-Quadrant Grid</td>
<td>A10.33</td>
</tr>
<tr>
<td><strong>Set B1</strong>: Algebra: Diagrams &amp; Equations</td>
<td>Activity 1: The Carnival</td>
<td>B1.1</td>
</tr>
<tr>
<td></td>
<td>Ind. Worksheet 1: Padre’s Pizza</td>
<td>B1.7</td>
</tr>
<tr>
<td></td>
<td>Ind. Worksheet 2: Choosing Equations &amp; Diagrams</td>
<td>B1.11</td>
</tr>
<tr>
<td><strong>Set A4</strong>: Number &amp; Operations: Long Division</td>
<td>Activity 1: Introducing the Standard Algorithm</td>
<td>A4.1</td>
</tr>
<tr>
<td></td>
<td>Activity 2: Extending the Standard Algorithm</td>
<td>A4.11</td>
</tr>
</tbody>
</table>

### UNIT 8

<table>
<thead>
<tr>
<th>Set, Strand &amp; Topic</th>
<th>Activity or Independent Worksheet</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>No supplement sets used</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
## Unit One Planner (Bridges Sessions)

**Note:** Sessions 1–3 have been omitted.

<table>
<thead>
<tr>
<th>SESSION 4</th>
<th>SESSION 5</th>
<th>SESSION 6</th>
<th>SESSION 7</th>
<th>SESSION 8</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Assessment</strong>&lt;br&gt;Unit One Pre-Assessment</td>
<td><strong>Problems &amp; Investigations</strong>&lt;br&gt;Discovering Patterns in Sequences of Cube Arrangements</td>
<td><strong>Problems &amp; Investigations</strong>&lt;br&gt;Using Patterns to Solve Problems</td>
<td><strong>Problems &amp; Investigations</strong>&lt;br&gt;Introducing the Area Model for Multiplication &amp; Division</td>
<td><strong>Problems &amp; Investigations</strong>&lt;br&gt;The Sharing &amp; Grouping Models for Division</td>
</tr>
<tr>
<td><strong>Assessment</strong>&lt;br&gt;Individual Interview (optional)</td>
<td><strong>Home Connection 2</strong></td>
<td><strong>Work Sample</strong></td>
<td><strong>Home Connection 3</strong></td>
<td><strong>Home Connection 4</strong></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>SESSION 9</th>
<th>SESSION 10</th>
<th>SESSION 11</th>
<th>SESSION 12</th>
<th>SESSION 13</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Problems &amp; Investigations</strong>&lt;br&gt;Prime &amp; Composite Numbers</td>
<td><strong>Problems &amp; Investigations</strong>&lt;br&gt;Introducing Add to 15 &amp; 15-Tac-Toe</td>
<td><strong>Problems &amp; Investigations</strong>&lt;br&gt;Prime Factorization</td>
<td><strong>Problems &amp; Investigations</strong>&lt;br&gt;Prime Factorization Number Riddles</td>
<td><strong>Problems &amp; Investigations</strong>&lt;br&gt;Order of Operations</td>
</tr>
<tr>
<td></td>
<td><strong>Home Connection 5</strong></td>
<td></td>
<td></td>
<td><strong>Home Connection 6</strong></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>SESSION 14</th>
<th>SESSION 15</th>
<th>SESSION 16</th>
<th>SESSION 17</th>
<th>SESSION 18</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Problems &amp; Investigations</strong>&lt;br&gt;Roll 5</td>
<td><strong>Work Places</strong>&lt;br&gt;1A 15-Tac-Toe 1B Roll 5</td>
<td><strong>Problems &amp; Investigations</strong>&lt;br&gt;Introducing Variables</td>
<td><strong>Problems &amp; Investigations</strong>&lt;br&gt;Using Variables</td>
<td><strong>Problems &amp; Investigations</strong>&lt;br&gt;Making Graphs &amp; T-Charts</td>
</tr>
<tr>
<td></td>
<td><strong>Home Connection 7</strong></td>
<td></td>
<td><strong>Home Connection 8</strong></td>
<td><strong>Work Sample</strong></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>SESSION 19</th>
<th>SESSION 20</th>
<th>SESSION 21</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Problems &amp; Investigations</strong>&lt;br&gt;Analyzing Data, Part 1 of 2</td>
<td><strong>Problems &amp; Investigations</strong>&lt;br&gt;Analyzing Data, Part 2 of 2</td>
<td><strong>Assessment</strong>&lt;br&gt;Unit One Post-Assessment</td>
</tr>
<tr>
<td><strong>Home Connection 9</strong></td>
<td><strong>Work Sample</strong></td>
<td><strong>Home Connection 10</strong></td>
</tr>
</tbody>
</table>
# Unit Two Planner (Bridges Sessions)

**Note:** No changes made.

<table>
<thead>
<tr>
<th>SESSION 1</th>
<th>SESSION 2</th>
<th>SESSION 3</th>
<th>SESSION 4</th>
<th>SESSION 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Home Connection 11</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SESSION 6</td>
<td>SESSION 7</td>
<td>SESSION 8</td>
<td>SESSION 9</td>
<td>SESSION 10</td>
</tr>
<tr>
<td></td>
<td>Home Connection 14</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SESSION 11</td>
<td>SESSION 12</td>
<td>SESSION 13</td>
<td>SESSION 14</td>
<td>SESSION 15</td>
</tr>
<tr>
<td>Problems &amp; Investigations Reviewing &amp; Evaluating Multiplication Strategies Work Sample</td>
<td>Problems &amp; Investigations An Extended Multiplication Story Problem</td>
<td>Problems &amp; Investigations Exploring Division with the Array Model Work Sample</td>
<td>Problems &amp; Investigations Division with Remainders Home Connection 18</td>
<td>Problems &amp; Investigations Quotients Win</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Home Connection 17</td>
</tr>
<tr>
<td>SESSION 16</td>
<td>SESSION 17</td>
<td>SESSION 18</td>
<td>SESSION 19</td>
<td>SESSION 20</td>
</tr>
<tr>
<td>Problems &amp; Investigations Writing Division Story Problems</td>
<td>Problems &amp; Investigations Solving One Another’s Division Story Problems with Sketches</td>
<td>Problems &amp; Investigations Division Showdown Home Connection 20</td>
<td>Work Places 2A Quotients Win 2B Division Showdown</td>
<td>Work Places Unit Two Work Places</td>
</tr>
<tr>
<td>Home Connection 19</td>
<td>Work Sample</td>
<td></td>
<td></td>
<td>Work Sample</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Home Connection 21</td>
</tr>
<tr>
<td>SESSION 21</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Assessment Unit Two Post-Assessment</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

© The Math Learning Center www.mathlearningcenter.org Bridges in Mathematics Grade 5 Supplement • 11
### Unit Three Planner (Bridges & CCSS Grade 5 Supplement Sets C1 & D2)

**Note:** Sessions 3, 4, 15, 17, 19 & 21 have been omitted to accommodate Supplement Sets C1 & D2.

<table>
<thead>
<tr>
<th>SESSION 1</th>
<th>SESSION 2</th>
<th>SESSION 5</th>
<th>SUPPLEMENT</th>
<th>SUPPLEMENT</th>
</tr>
</thead>
<tbody>
<tr>
<td>Problems &amp; Investigations&lt;br&gt;Which Is Bigger?&lt;br&gt;Work Sample</td>
<td>Problems &amp; Investigations&lt;br&gt;Dividing a Rectangle&lt;br&gt;Home Connection 22</td>
<td>Assessment&lt;br&gt;Unit Three Pre-Assessment</td>
<td>Supplement Set C1&lt;br&gt;Activity 1: Classifying Triangles&lt;br&gt;Homework&lt;br&gt;Supplement Set C1&lt;br&gt;Ind. Worksheet 1: More Geoboard Triangles</td>
<td>Supplement Set C1&lt;br&gt;Activity 2: Sorting &amp; Classifying Quadrilaterals&lt;br&gt;Homework&lt;br&gt;Supplement Set C1&lt;br&gt;Ind. Worksheet 2: Color &amp; Construct Triangles</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>SUPPLEMENT</th>
<th>SUPPLEMENT</th>
<th>SESSION 6</th>
<th>SESSION 7</th>
<th>SESSION 8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Supplement Set C1&lt;br&gt;Activity 3: Finding the Perimeter and Area of a Parallelogram&lt;br&gt;Homework&lt;br&gt;Supplement Set C1&lt;br&gt;Ind. Worksheet 3: Classifying Quadrilaterals</td>
<td>Supplement Set C1&lt;br&gt;Activity 4: Three Mathematical Ideas&lt;br&gt;Home Connection 24</td>
<td>Problems &amp; Investigations&lt;br&gt;Pattern Block Angles&lt;br&gt;Home Connection 24</td>
<td>Problems &amp; Investigations&lt;br&gt;Angle Measures Triangles &amp; Quadrilaterals&lt;br&gt;Home Connection 25</td>
<td>Problems &amp; Investigations&lt;br&gt;Sir Cumference and the Great Knight of Angleland</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>SESSION 9</th>
<th>SESSION 10</th>
<th>SESSION 11</th>
<th>SESSION 12</th>
<th>SESSION 13</th>
</tr>
</thead>
<tbody>
<tr>
<td>Problems &amp; Investigations&lt;br&gt;Angle Measure: From Pattern Blocks to Protractors&lt;br&gt;Work Sample</td>
<td>Problems &amp; Investigations&lt;br&gt;Parallels, Perpendiculars, and Angles&lt;br&gt;Home Connection 26</td>
<td>Problems &amp; Investigations&lt;br&gt;Congruence&lt;br&gt;Homework&lt;br&gt;Supplement Set C1&lt;br&gt;Ind. Worksheet 6: Ebony’s Quilt</td>
<td>Problems &amp; Investigations&lt;br&gt;Symmetry&lt;br&gt;Work Sample&lt;br&gt;Home Connection 27</td>
<td>Problems &amp; Investigations&lt;br&gt;Guess My Polygon</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>SESSION 14</th>
<th>SESSION 16</th>
<th>SESSION 18</th>
<th>SESSION 20</th>
<th>SUPPLEMENT</th>
</tr>
</thead>
<tbody>
<tr>
<td>Problems &amp; Investigations&lt;br&gt;Writing Polygon Riddles&lt;br&gt;Home Connection 28</td>
<td>Work Places&lt;br&gt;3A Area Bingo&lt;br&gt;3B Polygon Riddles&lt;br&gt;Work Sample</td>
<td>Problems &amp; Investigations&lt;br&gt;Building 3–Dimensional Figures&lt;br&gt;Work Sample</td>
<td>Problems &amp; Investigations&lt;br&gt;Volume&lt;br&gt;Supplement Set D2&lt;br&gt;Activity 1: Introducing Volume</td>
<td>Supplement Set D2&lt;br&gt;Activity 2: More Paper Boxes&lt;br&gt;Supplement Set D2&lt;br&gt;Ind. Worksheets 1 &amp; 2: Volume Review and The Camping Trip</td>
</tr>
</tbody>
</table>

### SESSION 22

<table>
<thead>
<tr>
<th>SUPPLEMENT</th>
<th>SUPPLEMENT</th>
</tr>
</thead>
<tbody>
<tr>
<td>Assessment&lt;br&gt;Unit 3 Post-Assessment</td>
<td>Supplement Set D2&lt;br&gt;Activity 6: Water for the Fish Tank Assessment</td>
</tr>
</tbody>
</table>
## Unit Four Planner (Bridges Sessions)

**Note:** No changes made.

<table>
<thead>
<tr>
<th>SESSION 1</th>
<th>SESSION 2</th>
<th>SESSION 3</th>
<th>SESSION 4</th>
<th>SESSION 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Assessment</td>
<td>Problems &amp; Investigations</td>
<td>Problems &amp; Investigations</td>
<td>Problems &amp; Investigations</td>
<td>Problems &amp; Investigations</td>
</tr>
<tr>
<td>Unit Four Pre-Assessment</td>
<td>Estimate &amp; Check</td>
<td>Multiplication Menus</td>
<td>Using Menus to Solve Long Division Problems</td>
<td>Making Menus &amp; Solving Problems</td>
</tr>
<tr>
<td></td>
<td>Home Connection 32</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>SESSION 6</th>
<th>SESSION 7</th>
<th>SESSION 8</th>
<th>SESSION 9</th>
<th>SESSION 10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water Conservation Division</td>
<td>Lowest Remainder Wins</td>
<td>4A Estimate &amp; Check 4B Lowest Remainder Wins</td>
<td>4C Measure &amp; Convert 4D Go for Zero</td>
<td>Same Dividend, Double the Divisor</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Home Connection 34</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>SESSION 11</th>
<th>SESSION 12</th>
<th>SESSION 13</th>
<th>SESSION 14</th>
<th>SESSION 15</th>
</tr>
</thead>
<tbody>
<tr>
<td>Problems &amp; Investigations</td>
<td>Problems &amp; Investigations</td>
<td>Problems &amp; Investigations</td>
<td>Problems &amp; Investigations</td>
<td>Problems &amp; Investigations</td>
</tr>
<tr>
<td>Making Fraction Kits</td>
<td>Race to Two &amp; Back</td>
<td>The Licorice &amp; Trail Mix Problems</td>
<td>Quilt Block Fractions</td>
<td>Fractions on the Geoboard</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Work Sample</td>
</tr>
<tr>
<td>Home Connection 36</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>SESSION 16</th>
<th>SESSION 17</th>
<th>SESSION 18</th>
<th>SESSION 19</th>
<th>SESSION 20</th>
</tr>
</thead>
<tbody>
<tr>
<td>Problems &amp; Investigations</td>
<td>Problems &amp; Investigations</td>
<td>Problems &amp; Investigations</td>
<td>Problems &amp; Investigations</td>
<td>Problems &amp; Investigations</td>
</tr>
<tr>
<td>Designing Quilt Blocks</td>
<td>Egg Carton Fractions</td>
<td>Name That Fraction</td>
<td>Combining Egg Carton Fractions</td>
<td>Fraction Story Problems</td>
</tr>
<tr>
<td>Work Sample</td>
<td></td>
<td></td>
<td></td>
<td>Work Sample</td>
</tr>
<tr>
<td>Home Connection 38</td>
<td></td>
<td></td>
<td></td>
<td>Home Connection 40</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>SESSION 21</th>
<th>SESSION 22</th>
<th>SESSION 23</th>
</tr>
</thead>
<tbody>
<tr>
<td>Work Places 4E Dozens of Eggs 4F Measuring to Fractions of an Inch</td>
<td>Work Places 4G Target Practice</td>
<td>Assessment Unit Four Post-Assessment</td>
</tr>
<tr>
<td>Home Connection 41</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
## Unit Six Planner (Bridges & CCSS Grade 5 Supplement Sets A6 & A11)

**Note:** Sessions 3 & 4 have been omitted to accommodate Supplement Sets A6 & A11. We recommend teaching Unit 6 before Unit 5.

<table>
<thead>
<tr>
<th>SESSION 1</th>
<th>SESSION 2</th>
<th>SUPPLEMENT</th>
<th>SUPPLEMENT</th>
<th>SESSION 5</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Assessment</strong>&lt;br&gt;Unit Six Pre-Assessment</td>
<td><strong>Problems &amp; Investigations</strong>&lt;br&gt;Fractions Are Quotients</td>
<td><strong>Supplement Set A6</strong>&lt;br&gt;Activity 1: Simplify &amp; Compare</td>
<td><strong>Supplement Set A6</strong>&lt;br&gt;Activity 2: Same-Sized Pieces</td>
<td><strong>Problems &amp; Investigations</strong>&lt;br&gt;Adding &amp; Subtracting Fractions, Part 1 of 2</td>
</tr>
<tr>
<td></td>
<td><strong>Work Sample</strong></td>
<td><strong>Homework Supplement Set A6</strong>&lt;br&gt;Ind. Worksheet 1: Using the Greatest Common Factor to Simplify Fractions</td>
<td><strong>Home Connection 50</strong></td>
<td><strong>Homework Supplement Set A6</strong>&lt;br&gt;Ind. Worksheet 2: Finding the Least Common Denominator</td>
</tr>
<tr>
<td></td>
<td><strong>Home Connection 49</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>SESSION 6</th>
<th>SESSION 7</th>
<th>SESSION 8</th>
<th>SESSION 9</th>
<th>SESSION 10</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Problems &amp; Investigations</strong>&lt;br&gt;Adding &amp; Subtracting Fractions, Part 2 of 2</td>
<td><strong>Work Places</strong>&lt;br&gt;6A Spin, Add &amp; Compare Fractions</td>
<td><strong>Problems &amp; Investigations</strong>&lt;br&gt;Shifting into Decimals: The Great Wall of Base Ten</td>
<td><strong>Problems &amp; Investigations</strong>&lt;br&gt;Modeling, Reading &amp; Comparing Decimals</td>
<td><strong>Problems &amp; Investigations</strong>&lt;br&gt;Fractions, Money, Decimals &amp; Division</td>
</tr>
<tr>
<td><strong>Work Sample</strong></td>
<td><strong>Homework Supplement Set A6</strong>&lt;br&gt;Ind. Worksheet 3: LCM &amp; GCF</td>
<td><strong>Home Connection 52</strong></td>
<td></td>
<td><strong>Home Connection 53</strong></td>
</tr>
<tr>
<td><strong>Home Connection 51</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>SESSION 11</th>
<th>SESSION 12</th>
<th>SESSION 13</th>
<th>SESSION 14</th>
<th>SESSION 15</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Problems &amp; Investigations</strong>&lt;br&gt;Thousandths and Ten Thousandths</td>
<td><strong>Problems &amp; Investigations</strong>&lt;br&gt;Decimal &amp; Fraction Equivalencies</td>
<td><strong>Problems &amp; Investigations</strong>&lt;br&gt;Decimals on a Number Line</td>
<td><strong>Problems &amp; Investigations</strong>&lt;br&gt;Adding &amp; Subtracting Decimals</td>
<td><strong>Problems &amp; Investigations</strong>&lt;br&gt;Modeling Percent</td>
</tr>
<tr>
<td><strong>Home Connection 54</strong></td>
<td><strong>Home Connection 55</strong></td>
<td></td>
<td><strong>Work Sample</strong></td>
<td><strong>Home Connection 56</strong></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>SESSION 16</th>
<th>SESSION 17</th>
<th>SESSION 18</th>
<th>SESSION 19</th>
<th>SUPPLEMENT</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Problems &amp; Investigations</strong>&lt;br&gt;The Number Line Game</td>
<td><strong>Work Places</strong>&lt;br&gt;6B Number Line Game&lt;br&gt;6C Roll &amp; Compare Decimals&lt;br&gt;6D Sporting Percentages (Challenge)</td>
<td><strong>Work Places</strong>&lt;br&gt;Unit 6 Work Places</td>
<td><strong>Assessment</strong>&lt;br&gt;Unit Six Post-Assessment</td>
<td><strong>Supplement Set A11</strong>&lt;br&gt;Activity 1: Multiplying by Powers of Ten</td>
</tr>
<tr>
<td></td>
<td><strong>Home Connection 57</strong></td>
<td><strong>Home Connection 58</strong></td>
<td></td>
<td><strong>Homework Supplement Set A11</strong>&lt;br&gt;Ind. Worksheet 1: Thinking about Tenths, Hundredths &amp; Thousandths</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>SUPPLEMENT</th>
<th>SUPPLEMENT</th>
<th>SUPPLEMENT</th>
<th>SUPPLEMENT</th>
<th>SUPPLEMENT</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Supplement Set A11</strong>&lt;br&gt;Activity 2: Dividing by Powers of Ten</td>
<td><strong>Supplement Set A11</strong>&lt;br&gt;Activity 3: Using Decimals to Calculate Sale Prices</td>
<td><strong>Supplement Set A11</strong>&lt;br&gt;Activity 4: Multiplying Decimals</td>
<td><strong>Supplement Set A11</strong>&lt;br&gt;Activity 5: Building a Deck, Using Partial Products &amp; Arrays for Decimal Multiplication</td>
<td><strong>Supplement Set A11</strong>&lt;br&gt;Activity 6: Multiplying Decimals More/Less</td>
</tr>
<tr>
<td><strong>Homework Supplement Set A11</strong>&lt;br&gt;Ind. Worksheet 2: Very Small &amp; Very Large Numbers in Context</td>
<td><strong>Homework Supplement Set A11</strong>&lt;br&gt;Ind. Worksheet 3: Multiplying &amp; Dividing by Powers of Ten</td>
<td><strong>Homework Supplement Set A11</strong>&lt;br&gt;Ind. Worksheet 4: Using Landmark Fractions &amp; Percents to Multiply by Decimals</td>
<td><strong>Homework Supplement Set A11</strong>&lt;br&gt;Ind. Worksheet 5: Multiplying Two Decimal Numbers</td>
<td><strong>Supplement Set A11</strong>&lt;br&gt;Ind. Worksheet 6: Symbols, Geometry &amp; Patterns in Numbers</td>
</tr>
</tbody>
</table>

© The Math Learning Center www.mathlearningcenter.org Bridges in Mathematics Grade 5 Supplement • 17
<table>
<thead>
<tr>
<th><strong>SUPPLEMENT</strong></th>
<th><strong>SUPPLEMENT</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>Supplement Set A11 Activity 7: Dividing Decimals with Money &amp; Menus</td>
<td>Supplement Set A11 Activity 8: Using Models &amp; Strategies to Divide with Decimals</td>
</tr>
<tr>
<td>Homework Supplement Set A11 Ind. Worksheet 6: Comparing &amp; Multiplying Fractions &amp; Decimals</td>
<td>Homework Supplement Set A11 Ind. Worksheet 7: Olympic Swimmers</td>
</tr>
</tbody>
</table>

**Note** Use Supplement Set A11, Ind. Worksheet 8: Olympic Track Star for additional practice with decimals.
Unit Five Replacement Planner (CCSS Grade 5 Supplement Sets A9 & A12)

**Note:** Unit 5 has been replaced with Supplement Sets A9 & A12. We recommend teaching Unit 5 after Unit 6.

<table>
<thead>
<tr>
<th>SUPPLEMENT</th>
<th>SUPPLEMENT</th>
<th>SUPPLEMENT</th>
<th>SUPPLEMENT</th>
<th>SUPPLEMENT</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>SUPPLEMENT</strong></td>
<td><strong>SUPPLEMENT</strong></td>
<td><strong>SUPPLEMENT</strong></td>
<td><strong>SUPPLEMENT</strong></td>
<td><strong>SUPPLEMENT</strong></td>
</tr>
<tr>
<td><strong>Homework</strong></td>
<td><strong>Homework</strong></td>
<td><strong>Homework</strong></td>
<td><strong>Homework</strong></td>
<td><strong>Homework</strong></td>
</tr>
<tr>
<td><strong>SUPPLEMENT</strong></td>
<td><strong>SUPPLEMENT</strong></td>
<td><strong>SUPPLEMENT</strong></td>
<td><strong>SUPPLEMENT</strong></td>
<td><strong>SUPPLEMENT</strong></td>
</tr>
<tr>
<td><strong>Homework</strong></td>
<td><strong>Homework</strong></td>
<td><strong>Homework</strong></td>
<td><strong>Homework</strong></td>
<td><strong>Homework</strong></td>
</tr>
<tr>
<td>Supplement Set A12 Activity 4: Dividing a Whole Number by a Fraction</td>
<td>Supplement Set A12 Activity 5: Sharing Stories</td>
<td>Supplement Set A12 Activity 6: Dividing a Fraction by a Whole Number</td>
<td>Supplement Set A12 Activity 7: The Division Poster Project</td>
<td>Supplement Set A12 Activity 8: Dividing Fractions &amp; Whole Numbers Post-Assessment</td>
</tr>
<tr>
<td><strong>SUPPLEMENT</strong></td>
<td><strong>SUPPLEMENT</strong></td>
<td><strong>SUPPLEMENT</strong></td>
<td><strong>SUPPLEMENT</strong></td>
<td><strong>SUPPLEMENT</strong></td>
</tr>
<tr>
<td><strong>Homework</strong></td>
<td><strong>Homework</strong></td>
<td><strong>Homework</strong></td>
<td><strong>Homework</strong></td>
<td></td>
</tr>
</tbody>
</table>
Unit Seven Planner (Bridges & CCSS Grade 5 Supplement Sets A4, A10 & B1)

**Note:** Session 15 has been omitted to accommodate Supplement Sets A4, A10 & B1.

<table>
<thead>
<tr>
<th>SESSION 1</th>
<th>SESSION 2</th>
<th>SESSION 3</th>
<th>SUPPLEMENT</th>
<th>SUPPLEMENT</th>
</tr>
</thead>
<tbody>
<tr>
<td>Home Connection 59</td>
<td></td>
<td>Home Connection 60</td>
<td></td>
<td>Home Connection 60</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>SUPPLEMENT</th>
<th>SESSION 4</th>
<th>SESSION 5</th>
<th>SESSION 6</th>
<th>SESSION 7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Homework: Supplement Set A10 Ind. Worksheet 3: Shapes on a 4-Quadrant Grid</td>
<td>Work Sample</td>
<td>Home Connection 61</td>
<td>Work Sample</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>SESSION 8</th>
<th>SESSION 9</th>
<th>SESSION 10</th>
<th>SESSION 11</th>
<th>SESSION 12</th>
</tr>
</thead>
<tbody>
<tr>
<td>Work Sample</td>
<td></td>
<td>Home Connection 63</td>
<td></td>
<td>Work Sample</td>
</tr>
<tr>
<td>Home Connection 62</td>
<td></td>
<td></td>
<td></td>
<td>Home Connection 64</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>SESSION 13</th>
<th>SESSION 14</th>
<th>SUPPLEMENT</th>
<th>SESSION 15</th>
<th>SUPPLEMENT</th>
</tr>
</thead>
<tbody>
<tr>
<td>Home Connection 65</td>
<td></td>
<td>Homework: Supplement Set B1 Ind. Worksheet 1: Padre’s Pizza</td>
<td>Home Connection 66</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

| SUPPLEMENT | |
|------------| |
| Supplement Set A4 Activity 2: Extending the Standard Algorithm | |
Unit Eight Planner (Bridges Sessions)

**Note:** No changes made.

<table>
<thead>
<tr>
<th>SESSION 1</th>
<th>SESSION 2</th>
<th>SESSION 3</th>
<th>SESSION 4</th>
<th>SESSION 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Problems &amp; Investigations</td>
<td>Problems &amp; Investigations</td>
<td>Problems &amp; Investigations</td>
<td>Problems &amp; Investigations</td>
<td>Problems &amp; Investigations</td>
</tr>
<tr>
<td>Reviewing Circles &amp; Angles</td>
<td>Making Cardstock</td>
<td>Introduction to Top Making</td>
<td>Exploring Tops Work Sample</td>
<td>Experiment 1: Tip Length</td>
</tr>
<tr>
<td></td>
<td>Compasses</td>
<td></td>
<td></td>
<td>Home Connection 67</td>
</tr>
<tr>
<td></td>
<td>Home Connection 66</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>SESSION 6</th>
<th>SESSION 7</th>
<th>SESSION 8</th>
<th>SESSION 9</th>
<th>SESSION 10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Problems &amp; Investigations</td>
<td>Problems &amp; Investigations</td>
<td>Problems &amp; Investigations</td>
<td>Problems &amp; Investigations</td>
<td>Problems &amp; Investigations</td>
</tr>
<tr>
<td>Analyzing the Data from</td>
<td>Experiment 2: Disk Mass</td>
<td>Analyzing the Data from</td>
<td>Design a Super Top</td>
<td>Experiments with Color &amp;</td>
</tr>
<tr>
<td>Experiment 1</td>
<td></td>
<td>Experiment 2</td>
<td>(Optional)</td>
<td>Designs</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Work Sample</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Home Connection 68</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>SESSION 11</th>
<th>SESSION 12</th>
</tr>
</thead>
<tbody>
<tr>
<td>Problems &amp; Investigations</td>
<td>Assessment</td>
</tr>
<tr>
<td>Experiments with Black &amp;</td>
<td>Unit Eight Post-</td>
</tr>
<tr>
<td>White</td>
<td>Assessment (Optional)</td>
</tr>
<tr>
<td>Home Connection 69</td>
<td></td>
</tr>
</tbody>
</table>
# Grade 5 CCSS Supplement Materials List

<table>
<thead>
<tr>
<th>MANIPULATIVES</th>
<th>A4</th>
<th>A6</th>
<th>A9</th>
<th>A10</th>
<th>A11</th>
<th>A12</th>
<th>B1</th>
<th>C1</th>
<th>D2</th>
</tr>
</thead>
<tbody>
<tr>
<td>T400T Color Tile (3 sets of 400)*</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LUR36 Red linear pieces (5 sets)*</td>
<td>√</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LU Black linear pieces (1 set)*</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PGT Base 10 pieces (class set)*</td>
<td></td>
<td>√</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>OH10 Overhead Base 10 pieces*</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>G15B Clear geoboards &amp; bands (class set plus 1)*</td>
<td>√</td>
<td></td>
<td></td>
<td>√</td>
<td></td>
<td>√</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>More/Less cubes (15)*</td>
<td>√</td>
<td></td>
<td></td>
<td>√</td>
<td></td>
<td>√</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>D45NUM Dice numbered 1–6 and 4–9 (15 of each)*</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CW75 Blank dice or wood cubes (15)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SPOH-TEMP Overhead double spinner overlay*</td>
<td>√</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SPOHS Transparent spinner overlays (15)*</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>M400 Game markers*</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>RLC Rulers that show inches &amp; centimeters (class set)*</td>
<td>√</td>
<td></td>
<td></td>
<td>√</td>
<td></td>
<td>√</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PRO180 Protractors (class set)*</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>BWRC Word Resource Cards*</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CW-1CM Centimeter Cubes (2 buckets of 1,000)*</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MONVPA Money Value Pieces (half class set)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>BOTQT Capacity Containers</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>BSJ Student Math Journals*</td>
<td>√</td>
<td></td>
<td></td>
<td>√</td>
<td></td>
<td>√</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>T101 Calculators</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

All manipulatives available from Math Learning Center. Those items marked with an asterisk are included in the Bridges Grade 5 Package.

<table>
<thead>
<tr>
<th>GENERAL MATERIALS (PROVIDED BY THE TEACHER)</th>
<th>A4</th>
<th>A6</th>
<th>A9</th>
<th>A10</th>
<th>A11</th>
<th>A12</th>
<th>B1</th>
<th>C1</th>
<th>D2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Computers/Internet Access</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Computer projection equipment</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Overhead or document camera</td>
<td>√</td>
<td>√</td>
<td>√</td>
<td>√</td>
<td></td>
<td>√</td>
<td>√</td>
<td>√</td>
<td>√</td>
</tr>
<tr>
<td>Blank overhead transparencies if you are using an overhead projector rather than a doc camera</td>
<td>4</td>
<td>5</td>
<td>10</td>
<td>4</td>
<td>9</td>
<td>4</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>8.5” x 11” card stock, sheets per student</td>
<td></td>
<td>2</td>
<td></td>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8.5” x 11” colored copy paper, sheets per student</td>
<td></td>
<td></td>
<td></td>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8.5” x 11” lined or grid paper, sheets per student</td>
<td>2</td>
<td>3</td>
<td></td>
<td>1</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9” x 12” and 12” x 18” construction paper</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Chart Paper</td>
<td>√</td>
<td>√</td>
<td></td>
<td>√</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2” x 18” drawing paper</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Student name tags</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Student whiteboards, erasers, pens</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 1/2” x 2” sticky notes</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3” x 5” index cards (3 per student)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Overhead pens (black, blue, red)</td>
<td>√</td>
<td>√</td>
<td></td>
<td>⊖</td>
<td>√</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Scissors, class set</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Transparent tape (several rolls)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Masking Tape</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Regular pencils</td>
<td>√</td>
<td>√</td>
<td>√</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Opt means the item is optional.
### Grade 5 CCSS Supplement Materials List (cont.)

<table>
<thead>
<tr>
<th>GENERAL MATERIALS (PROVIDED BY THE TEACHER)</th>
<th>A4</th>
<th>A6</th>
<th>A9</th>
<th>A10</th>
<th>A11</th>
<th>A12</th>
<th>B1</th>
<th>C1</th>
<th>D2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Colored pencils, crayons, felt markers</td>
<td></td>
<td></td>
<td>√</td>
<td>√</td>
<td>√</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Clean, empty half-gallon milk carton</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Opt</td>
</tr>
<tr>
<td>*2 one-quart bottles from the Grade 5 Number Corner Kit</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Opt</td>
</tr>
<tr>
<td>*1-cup and 2-cup liquid measuring cups from the Grade 5 Number Corner Kit</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Opt</td>
</tr>
<tr>
<td>Small bottle of food coloring</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Opt</td>
</tr>
<tr>
<td>Empty clean plastic bowls or containers in the following numbers: 4 two-cup containers, 8 one-cup containers, 16 half-cup containers</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Opt</td>
</tr>
<tr>
<td><em>Counting on Frank</em>, by Rod Clement</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Opt</td>
</tr>
</tbody>
</table>
GRADE 5 – UNIT 1
CCSS SUPPLEMENT ACTIVITIES & INDEPENDENT WORKSHEETS

No Supplements Used
GRADE 5 – UNIT 2

CCSS SUPPLEMENT ACTIVITIES & INDEPENDENT WORKSHEETS

No Supplements Used
Set C1: Geometry: Triangles & Quadrilaterals
- Activity 1: Classifying Triangles ............................................ C1.1
- Ind. Worksheet 1: More Geoboard Triangles .......................... C1.43
- Activity 2: Sorting & Classifying Quadrilaterals ...................... C1.13
- Ind. Worksheet 2: Color & Construct Triangles ...................... C1.45
- Activity 3: Finding the Perimeter & Area of a Parallelogram ........ C1.25
- Ind. Worksheet 3: Classifying Quadrilaterals ......................... C1.47
- Activity 4: Three Mathematical Ideas .................................... C1.35
- Ind. Worksheet 4: Quad Construction ................................. C1.51
- Ind. Worksheet 5: Perimeter & Area Puzzles ......................... C1.53
- Ind. Worksheet 6: Ebony’s Quilt ......................................... C1.55

Set D2: Measurement: Volume
- Activity 1: Introducing Volume ............................................. D2.1
- Activity 2: More Paper Boxes ........................................... D2.7
- Ind. Worksheet 1: Volume Review ....................................... D2.35
- Ind. Worksheet 2: The Camping Trip ................................... D2.39
- Activity 3: Fish Tank Volume ............................................. D2.13
- Ind. Worksheet 3: Tank Volume ......................................... D2.41
- Activity 4: Number of Fish ............................................. D2.19
- Ind. Worksheet 4: Third Tank Needs Fish ......................... D2.43
- Activity 5: Fish Tank Plants with a Line Plot ....................... D2.25
- Ind. Worksheet 5: Water for the Fish Tank ....................... D2.45
- Activity 6: Volume & Line Plot Assessment ....................... D2.33
Set C1 ★ Activity 1

Classifying Triangles

Overview
Students build and record four different triangles on their geoboards. Then they classify their triangles, first by angle size and then by side length.

Skills & Concepts
★ classify triangles by the length of their sides as either scalene, isosceles, or equilateral
★ classify triangles by the size of their angles as either acute, obtuse, or right
★ classify angles as either right, acute, or obtuse

You’ll need
★ Triangles Record Sheet (page C1.5, run a class set plus a few extra and one copy on a transparency)
★ Types of Triangles (page C1.6, run one copy on a transparency)
★ overhead geoboard
★ class set of geoboards and rubber bands
★ class set of rulers
★ a piece of paper to mask parts of the overhead
★ access to protractors
★ Word Resource Cards: acute angle, obtuse angle, right angle (pages D6.7–D6.12, run 1 copy back to back on cardstock, cut out each card. See Advance Preparation)

Advance Preparation Post the Word Resource Cards where all the students can see them clearly before you conduct this activity.

Instructions for Classifying Triangles
1. Ask students to get out their rulers and pencils. Then give them each a geoboard and a copy of the Triangles Record Sheet. Explain that they are going to make and record 4 different types of triangles today. Demonstrate by making a triangle on a geoboard at the overhead. If necessary, review any guidelines you have established with the class for handling the rubber bands carefully. Then copy your triangle onto the Triangles Record Sheet transparency. Solicit advice from students about how to do this carefully and accurately as you are working.
2. When students understand what to do, pass out the rubber bands and let them get started. Remind them to make 4 different triangles. Encourage them to make triangles that are different than the one you made, and different from the ones their neighbors are making. Circulate as they are working to talk with them about their triangles. What kinds of angles do they notice as they create their triangles? Can they point out acute, obtuse, and/or right angles in their work?

3. When most students have finished, reconvene the class. Explain that they are going to classify by type, and record, the triangles they have just created. Show just the top portion of Types of Triangles at the overhead.

4. Read and discuss the information with the class. Ask volunteers to work with the support of the pictures on the Word Resource Cards to describe each type of angle and label an example of each on the overhead. Then have the students help you classify the triangle you made on your geoboard.

   **Teacher**  What kind of triangle did I make when I introduced this activity? I’ll hold up my geoboard so you can see it while you look at the different types of triangles on the overhead. Pair-share with the person next to you, and raise your hand when you have an idea.

   **Students**  I think it’s an acute triangle because it’s so skinny. It’s none of those because it doesn’t look like any of the triangles on the overhead. I’m almost sure the angle at the bottom is a right angle. I think it’s a right triangle. Can we test it out? Let’s see if a square pattern block will fit in that corner.

You may have to help students understand that a triangle doesn’t have to look exactly like the ones on the overhead to fit into one of the three categories. If necessary, build several more triangles on your board and have the students work together to classify them.
Activity 1  Classifying Triangles (cont.)

5. When students understand what to do, have them work in pairs to classify the triangles on their record sheets by angle size. Ask them to record the classification on the first line in the box below each triangle.

6. As students finish their work, have them confer with others nearby. If there are disagreements, encourage students to work together to resolve them. How can they be certain an angle is acute, right, or obtuse?

7. When most students have finished, reconvene the class and display the other half of the Triangle Types overhead. Read and discuss the information with students.

8. Ask students to help you classify the triangle you made on your geoboard by the lengths of its sides. Remind them that a triangle doesn't have to look exactly like one of the examples on the overhead to fit one of the categories. When they have come to agreement, record the information on your record sheet.

9. Have students work in pairs to classify their own triangles by side length and record the information on their sheets. Keep the Types of Triangle overhead posted for their reference.
Activity 1  Classifying Triangles (cont.)

10. A time allows, ask students to share and compare some of the triangles they made. Let them know that it is, in fact, impossible to create an equilateral triangle on this geoboard. If any of the students believe they have created an equilateral triangle, have them share it with the class, and work together to measure the sides very carefully. While the side lengths may be very close, they will not be equal.

INDEPENDENT WORKSHEET

Use Set C1 Independent Worksheets 1 and 2 to provide students with more practice identifying, describing, and classifying triangles by angle size and side length. These sheets also ask students to draw triangles from given information about sides and angles.
## Triangles Record Sheet

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>![Triangle 1]</td>
<td>![Triangle 2]</td>
</tr>
<tr>
<td>![Triangle 3]</td>
<td>![Triangle 4]</td>
</tr>
<tr>
<td>![Triangle 5]</td>
<td>![Triangle 6]</td>
</tr>
<tr>
<td>![Triangle 7]</td>
<td>![Triangle 8]</td>
</tr>
</tbody>
</table>

**NAME**

**DATE**
## Types of Triangles

1. You can classify triangles by the size of their angles.

   **Acute Triangle**
   All 3 angles are acute.

   **Right Triangle**
   One of the angles is a right angle.

   **Obtuse Triangle**
   One of the angles is obtuse.

2. You can also classify triangles by the length of their sides.

   **Isosceles Triangle**
   Two sides are the same length.

   **Scalene Triangle**
   Each side is a different length.

   **Equilateral Triangle**
   Each side is the same length.

   Are any of the triangles you made on the geoboard equilaterals? Can you make an equilateral triangle on a geoboard?
acute angle
measures less than 90°
acute angle: an angle that has a measure less than 90°.
obtuse angle
measures more than 90°
more than 90° and less than 180°

obtuse angle: an angle that has a measure
right angle

measures exactly 90°
right angle: an angle that has a 90° measure
More Geoboard Triangles

Remember that you can classify and describe triangles in two different ways:

• by the size of their angles

- **Acute Triangle**
  All 3 angles are acute.

- **Right Triangle**
  One of the angles is a right angle.

- **Obtuse Triangle**
  One of the angles is obtuse.

• by the length of their sides

- **Isosceles Triangle**
  Two sides are the same length.

- **Scalene Triangle**
  Each side is a different length.

- **Equilateral Triangle**
  All 3 sides are the same length.

Follow the instructions below each geoboard to draw some different triangles.

**Hint** Build your triangles on a geoboard first. Then copy them onto the paper.

1 A Right Triangle

2 An Isosceles Triangle

3 An Acute Triangle

(Continued on back.)
**Independent Worksheet 1**  More Geoboard Triangles (cont.)

4  An Obtuse Triangle

5  A Scalene Triangle

6  A Right Triangle that is also Isosceles

7  A Right Triangle that is also Scalene

8  An Obtuse Triangle that is also Isosceles

9  A Scalene Triangle that is not Obtuse

**CHALLENGE**

10  Dana says it is impossible to draw a right triangle that is also acute. Do you agree with her? Why or why not? Use the geoboards below to test your ideas. Explain your ideas in writing.
Set C1 ★ Activity 2

Sorting & Classifying Quadrilaterals

Overview
Students review what they have learned about quadrilaterals, and use the information to sort and classify quadrilaterals in a variety of ways.

Skills & Concepts
★ classify quadrilaterals
★ measure length with accuracy

You’ll need
★ Different Kinds of Quadrilaterals (page C1.19, run a copy on a transparency)
★ Sorting Quadrilaterals (page C1.20, run a copy on a transparency)
★ Paper Quadrilaterals (page C1.21, run a half-class set plus a few extra)
★ Venn Diagram Mat (page C1.22, run a half-class set)
★ The Logic of Quadrilaterals (page A1.23, optional, run a class set)
★ paper to mask parts of the overhead and overhead pens
★ class sets of scissors, rulers and protractors

Instructions for Sorting & Classifying Quadrilaterals
1. Write the word *quadrilateral* on the board or overhead. Ask students to pair-share what they know about this term right now. Then invite a few volunteers to share their ideas with the class. If it doesn’t emerge from the group, solicit agreement that a quadrilateral is a 4-sided polygon. Then work with student input to list several examples of different quadrilaterals.

2. Explain that the class is going to do some more work with quadrilaterals today. Display the top portion of Different Kinds of Quadrilaterals on the overhead. Read and discuss the name and description of each shape with students. Here are some questions you might pose as you review the terms with the class. Encourage students to use the information on the overhead as they formulate their answers.
   - What is the difference between a rhombus and a square?
   - Why do people say that a square is a special kind of rectangle?
   - Would it be fair to say that a square is a special kind of rhombus? Why?
   - Is a trapezoid also a parallelogram? Why or why not? (No, because it only has 1 pair of parallel sides.)
   - Why is a rhombus classified as a parallelogram? (Because it has 2 pairs of parallel sides opposite each other.)
   - Is a rhombus also a kite? Why or why not? (Yes, because it has two pairs of adjacent sides that are congruent; in fact, all 4 of its sides are congruent.)
   - Are there any other quadrilaterals that could be called kites? Which one(s), and why? (A square is also a kite because it has two pairs of adjacent sides that are congruent.)
   - Which one of these shapes could be given the most names? Why? (A square, because it is also a quadrilateral, a kite, a parallelogram, a rectangle, and a rhombus!)

© The Math Learning Center

Bridges in Mathematics Grade 5 Supplement • C1.13
### Different Kinds of Quadrilaterals

A Quadrilateral is any polygon with 4 sides.

<table>
<thead>
<tr>
<th><strong>Trapezoid</strong></th>
<th><strong>Parallelogram</strong></th>
<th><strong>Rectangle</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>a quadrilateral with exactly 1 pair of parallel sides</td>
<td>a quadrilateral with 2 pairs of parallel sides opposite each other</td>
<td>a parallelogram with 4 right angles</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>Rhombus</strong></th>
<th><strong>Square</strong></th>
<th><strong>Kite</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>a parallelogram with 4 congruent sides</td>
<td>a parallelogram with 4 congruent sides and 4 right angles</td>
<td>a quadrilateral with two pairs of adjacent sides that are congruent</td>
</tr>
</tbody>
</table>

3. Display the bottom portion of the overhead, and have students pair-share their responses to all five questions. Ask them to jot their answers down on a piece of scratch paper, and be prepared to explain and justify each. After a minute or two, reconvene the class. Invite a different volunteer to answer and explain his or her response to each question.

<table>
<thead>
<tr>
<th><strong>True or false?</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>1. This shape is a quadrilateral.</td>
</tr>
<tr>
<td>2. This shape is a trapezoid.</td>
</tr>
<tr>
<td>3. This shape is a rhombus.</td>
</tr>
<tr>
<td>4. This shape is a parallelogram.</td>
</tr>
<tr>
<td>5. This shape is a rectangle.</td>
</tr>
</tbody>
</table>

4. Next, ask students if any of the other quadrilateral names on the list applies to the shape at the bottom of the overhead. The shape is a rectangle, but it can also be called a quadrilateral and a parallelogram. It cannot be called a trapezoid or a rhombus.

- Can it be called a square or a kite? Why or why not? (Neither, because it does not have 4 congruent sides, nor does it have congruent sides that are adjacent to one another.)
- Which of the names describes the shape the most exactly and specifically? Why? (Rectangle, because a quadrilateral could be any 4-sided figure, and a parallelogram doesn't have to have 4 right angles.)

5. Now explain that the students are going to work in pairs to label and cut out a set of paper quadrilaterals. They will be sorting these quadrilaterals in a few minutes, but their first task is to label each with the name that describes it most exactly and specifically. Have students pair up and get out their scissors. They may also need rulers and protractors because they will probably have to measure the angles and side lengths of some of the shapes to identify them accurately.

Give each pair a copy of the Paper Quadrilaterals sheet. Ask them to cut it in half so each partner can label and cut out half the shapes in the set.
6. Once students understand the labeling and cutting procedures, have them go to work. Leave the Quadrilaterals overhead on display for their reference. Circulate to provide assistance as needed, but encourage students to help their partners and confirm their answers with other pairs nearby.

7. When most students have finished labeling and cutting out their shapes, confirm the name of each with the class. One simple way to do this is to have volunteers list the letters that belong in each shape group as you record at the overhead.
8. Next, give each student pair a Venn Diagram Mat, and explain that they are going to work together to sort their shapes in a variety of ways. Place the first prompt at the top of the Sorting Quadrilaterals overhead on display.

Read the prompt with the class, and ask students to sort their shapes onto the mat, quadrilaterals in one circle and trapezoids in the other. If there are any shapes that qualify as both quadrilaterals and trapezoids, ask students to place them between the circles, at the intersection of the two sets. If there are shapes that don’t fit either description, ask students to place them off to one side.

9. Encourage students to share and compare their results with other pairs nearby. When most pairs have finished, call on volunteers to share and explain their results. You may want to sketch a Venn diagram on the overhead and invite volunteers to sort their shapes for the class to see. You can also ask students to examine the speakers’ work from where they are sitting, or stand if necessary.

   **Students**  There are only 4 trapezoids, and they had to go in the middle because they are also quadrilaterals.

   All the shapes went on the mat because they all have 4 sides. The ones in the middle are quadrilaterals and trapezoids.

10. Repeat steps 8 and 9 as you display each of the other sorting prompts on the overhead one by one. Some of the prompts are more challenging than others, and may result in lively discussion and debate.
Extension

- Give students each a copy of The Logic of Quadrilaterals. The diagram on this sheet illustrates the relationships between the various quadrilaterals in a very succinct way. Students are asked to label each of the shapes, and then answer a series of questions designed to help them think about how the shapes have been placed in relation to one another, and why. After reviewing the instructions together, have students complete the sheet independently. Then discuss it as a group. (There is a copy of the diagram at the bottom of the Sorting Quadrilaterals overhead you can use to focus and direct the discussion.)
Activity 2 Sorting & Classifying Quadrilaterals (cont.)

INDEPENDENT WORKSHEET

Use Set C1 Independent Worksheets 3 and 4 to provide students with more practice classifying and drawing quadrilaterals from information given about sides and angles.
Different Kinds of Quadrilaterals

A Quadrilateral is any polygon with 4 sides

<table>
<thead>
<tr>
<th>Trapezoid</th>
<th>Parallelogram</th>
<th>Rectangle</th>
</tr>
</thead>
<tbody>
<tr>
<td>a quadrilateral with exactly 1 pair of parallel sides</td>
<td>a quadrilateral with 2 pairs of parallel sides opposite each other</td>
<td>a parallelogram with 4 right angles</td>
</tr>
<tr>
<td>Rhombus</td>
<td>Square</td>
<td>Kite</td>
</tr>
<tr>
<td>a parallelogram with 4 congruent sides</td>
<td>a parallelogram with 4 congruent sides and 4 right angles</td>
<td>a quadrilateral with two pairs of adjacent sides that are congruent</td>
</tr>
</tbody>
</table>

True or false?

1. This shape is a quadrilateral.
2. This shape is a trapezoid.
3. This shape is a rhombus.
4. This shape is a parallelogram.
5. This shape is a rectangle.
Sorting Quadrilaterals

1 Quadrilaterals/Trapezoids
2 Trapezoids/Parallelograms
3 Parallelograms/Rectangles
4 Rectangles/Rhombuses
5 Kites/Rectangles
6 Kites/Parallelograms
Paper Quadrilaterals

Label each quadrilateral with the most specific name you can find for it. Then cut out the shapes.
The Logic of Quadrilaterals

1. Label each shape in this diagram with the name that describes it most exactly.

2. Why is the trapezoid inside the quadrilateral but outside the parallelogram?

3. Why are there a rhombus and a rectangle inside the parallelogram?

4. Why are there two squares, one inside the rhombus and one inside the rectangle?

5. Write at least 2 other observations to explain why the shapes in this diagram have been placed where they are in relation to each other.
You can classify triangles by the size of their angles,

- **Acute Triangle**: All 3 angles are acute.
- **Right Triangle**: One of the angles is a right angle.
- **Obtuse Triangle**: One of the angles is obtuse.

1. Look at the triangles below. Color:
   - the acute triangles green.
   - the right triangles red.
   - the obtuse triangles orange.

   **Hint**: Use the corner of a piece of paper, a tile, or a square pattern block to help test the angles. Some of these triangles might fool you!

2. Follow the instructions below to draw your own triangles.

   - **a** Draw a right triangle with no congruent sides.
   - **b** Draw an acute triangle with 3 congruent sides.
   - **c** Draw an obtuse triangle with 2 congruent sides.
You can also classify triangles by the length of their sides.

<table>
<thead>
<tr>
<th>Triangle</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Isosceles Triangle</strong></td>
<td>Two sides are the same length.</td>
</tr>
<tr>
<td><strong>Scalene Triangle</strong></td>
<td>Each side is a different length.</td>
</tr>
<tr>
<td><strong>Equilateral Triangle</strong></td>
<td>All 3 sides are the same length.</td>
</tr>
</tbody>
</table>

3 Look at the triangles below. Color:
- the isosceles triangles purple.
- the scalene triangles yellow.
- the equilateral triangles blue.

**Hint** If you are not sure whether the side lengths are equal or not, use your ruler to help. Measure to the nearest half inch, or even the nearest centimeter.

4 Draw an isosceles triangle that is also a right triangle.

5 Draw a scalene triangle that is not an obtuse triangle.

6 Measure and label this triangle to show the length of each side and the measure of each angle.

7 Is the triangle in problem 6 acute, right, or obtuse? Is it isosceles, scalene, or equilateral? How do you know?
Finding the Perimeter & Area of a Parallelogram

Overview
Students find the perimeter and area of an index card, then cut the card and tape the two resulting pieces together to form a parallelogram that is not a rectangle. Then they find the perimeter and area of the parallelogram. As they do so, they discover that multiplying the length of one side by the other does not yield the area of a non-rectangular parallelogram. After students investigate further by creating two more parallelograms, the teacher shares the formula for finding the area of a parallelogram, and asks the class to explain and apply it.

Skills & Concepts
★ classify quadrilaterals
★ determine the formula for the area of a parallelogram by relating it to the area of a rectangle
★ use formulas to determine the perimeters and areas of rectangles and parallelograms
★ use appropriate tools and units to measure objects to the precision of one-eighth inch

You’ll need
★ Start with a Rectangle (page C1.30, run a copy on a transparency)
★ Square Inch Grid Paper (page C1.31, run several class sets and one copy on a transparency)
★ Finding the Area of Parallelograms (pages C1.32 and C1.33, run a class set)
★ paper to mask parts of the overhead
★ overhead pens
★ 3” x 5” index cards or pieces of construction paper, 3 per student
★ class set of rulers
★ scissors
★ several rolls of scotch tape

Instructions for Finding the Perimeter & Area of a Parallelogram
1. Write the words perimeter and area on the board. Have students pair-share the definition of each term, and then ask volunteers to share their definitions with the class. Briefly review the formulas for finding the perimeter ($2l + 2w$) and area ($l \times w$) of a rectangle, and give students each an index card. Ask students to measure the length and the width of the index card in inches, and use the information to find its perimeter and the area. Have them use a piece of scratch paper or the card itself if they need to do any writing as they determine these measurements.

2. When most students have finished, display just the first instruction on the Start with a Rectangle overhead, and work with input from the class to record the perimeter and area of the index card. Then reveal the second task on the overhead. Write 3” in the blank as you read the instruction with the class, and give students time to measure and mark their cards as specified. Ask them to be as precise as possible in their measurements. Show the rest of the tasks on the overhead one by one. Read each task with the class and give students time to complete it before moving on to the next. Take time to discuss each question, and record the answers on the overhead. Ask students to be certain they have formed a parallelogram that is not a rectangle before they use any scotch tape.
Activity 3  Finding the Perimeter & Area of a Parallelogram (cont.)

Start with a Rectangle

1. Find the perimeter of your rectangle in inches. Find the area of your rectangle in square inches.

Perimeter = __________  
Area = __________

2. Measure over 2" along the top of your rectangle and make a small mark.

3. Draw a diagonal line from the lower left-hand corner to the mark.

4. Cut along the line. What 2 shapes do you have now? How do you know?

right isosceles triangle, trapezoid

5. Combine the 2 shapes to make a parallelogram that is not a rectangle. Tape the edges together.

6. Find the perimeter of your parallelogram to the nearest eighth of an inch. Find the area of your parallelogram in square inches.

Perimeter = __________  
Area = __________

3. The last question on the overhead asks students to find the perimeter and area of the parallelogram they formed when they cut and taped the index card. When you reach this point, make square-inch grid paper available, and give students some time to investigate at their tables. Some may believe that the area is still 15 square inches because they didn't add anything or take anything away when they formed their parallelogram. Press them to find a way to prove this, using the grid paper or some other method. Other students may need to trace the parallelogram onto the grid paper and count the squares and triangles to discover that the area has remained the same, even though the perimeter has changed.

4. When most students have found the perimeter and area of the parallelogram, reconvene the class. Ask volunteers to share their results and strategies. Most will likely report that the perimeter is 18 1/2 inches, and the area is 15 square inches. Here are some questions to pose during the discussion:

• Is the perimeter of the parallelogram the same as the perimeter of the original rectangle? Why or why not?
• Is the area the same? Why or why not?
• Does the formula for finding the perimeter of a rectangle still work with this parallelogram?
• Does the formula for finding the area of a rectangle help you find the area of the parallelogram? If so, how? If not, why?

Students  The perimeter changed when we made the card into a parallelogram. It was 3 by 5, so the perimeter was 16 inches. Now it’s about 4 1/4 inches along the diagonal side and still 5 inches along the top. Two times 5 is 10, and two times 4 1/4 is 8 1/2, so that’s 18 1/2 inches now instead of 16. When you cut it on the diagonal like that, it definitely makes the sides longer.

Students  It’s still 15 square inches for the area, though. We traced it on the grid paper and counted the squares and triangles. It came out to be exactly 15 square inches.
Activity 3 Finding the Perimeter & Area of a Parallelogram (cont.)

You can't use the regular formula to find the area of the parallelogram. If you multiply $4 \frac{1}{4}$ times 5, it's more than 20 square inches. But you can see that the area is really 15 square inches, not 20 square inches.

I know one-fourth is .25, so I put in $5 \times 4.25$ on my calculator. It came out to be 21.25. That's $21 \frac{1}{4}$ square inches, but the parallelogram is really only 15 square inches.

5. Work with students' input to summarize their findings by sketching the rectangle and the parallelogram on the whiteboard and recording the perimeter and area of each.

6. Erase the overhead. Give students each another index card. Repeat the process a second time, but have them measure and mark over 2 inches instead of 3 inches this time. When they cut along the line, they will discover that they have formed a right scalene triangle and a trapezoid. Have them combine these two shapes to make a second parallelogram, and find the perimeter and area of this figure. Invite a couple of volunteers to trace their new parallelograms on the Square Inch Grid overhead and share their strategies for determining the area.

Toby's Strategy

Eric's Strategy

Toby  I just imagined cutting off the triangle at this end and sliding it over to the other side. You can see it will still be 15 square inches.

Eric  I surrounded the triangle at this end with a rectangle. That rectangle is 6, so the triangle is 3 square inches. If you do that with the triangles at both ends, and then add their areas to the square in the middle, it comes out to be $3 + 9 + 3$, and that's 15 square inches.
7. Summarize students' findings on the whiteboard. Then give them another index card, erase the overhead, and repeat the process once more. This time, have students measure and mark one inch over along the top of the card, draw the diagonal, make the cut, identify the two shapes that result (a right scalene triangle and a trapezoid), combine the two shapes to make a parallelogram, and find the perimeter and area. Invite a couple of volunteers to trace their parallelograms and share their strategies for finding the area.

8. Summarize the third set of findings on the whiteboard, and ask students to share any observations they can make. How and why did the perimeter change from one parallelogram to the next? Why did the area remain the same each time?

9. Display the information at the bottom of the overhead, which gives the formula for finding the area of a parallelogram that is not a rectangle. Ask students to discuss and explain how the formula works, based on their experiences during this activity.

10. Give students each a copy of Finding the Area of Parallelograms. Review the instructions on both sheets with the class. When students understand what to do, let them get started. Plan to assign unfinished work as homework or seatwork the following day.
Finding the Area of Parallelograms page 1 of 2

The height (h) of a parallelogram tells how far one side is from its opposite side. The height of a parallelogram must be perpendicular to the base (b) of the parallelogram.

1. Use the letter h to label the height of each parallelogram below. Use the letter b to label the base. If the height is not shown, use your ruler to draw it in, and then label it.

2. To find the area of a parallelogram, multiply base times height. Try it for yourself. Multiply the two measurements. Is the answer correct? Use a labeled sketch, numbers, and words to explain.

Finding the Area of Parallelograms page 2 of 2

Here are the formulas for the area of a rectangle and the area of a parallelogram.

- The area of a rectangle = length × width or l × w or lw.
- The area of a parallelogram = base × height or b × h or bh.

3. Find the area of each figure below. Use the formulas. Show your work.

4. For each of the parallelograms below:
   - draw in the height.
   - measure and label the height and the base to the nearest centimeter.
   - find and record the area and show your work.

NAME DATE
Start with a Rectangle

1 Find the perimeter of your rectangle in inches. Find the area of your rectangle in square inches.

Perimeter = __________                    Area = __________

2 Measure over _____ along the top of your rectangle and make a small mark.

3 Draw a diagonal line from the lower left-hand corner to the mark.

4 Cut along the line. What 2 shapes do you have now? How do you know?

5 Combine the 2 shapes to make a parallelogram that is not a rectangle. Tape the edges together.

6 Find the perimeter of your parallelogram to the nearest eighth of an inch. Find the area of your parallelogram in square inches.

Perimeter = __________                    Area = __________

7 Here is the formula for finding the area of a parallelogram that is not a rectangle. Explain how and why it works.

Area of parallelogram = base × height or bh
Square Inch Grid Paper
Finding the Area of Parallelograms page 1 of 2

The height \((h)\) of a parallelogram tells how far one side is from its opposite side. The height of a parallelogram must be perpendicular to the base \((b)\) of the parallelogram.

1. Use the letter \(h\) to label the height of each parallelogram below. Use the letter \(b\) to label the base. If the height is not shown, use your ruler to draw it in, and then label it.

2. To find the area of a parallelogram, multiply base times height. Try it for yourself. Measure the base and the height of the parallelogram below in centimeters. Multiply the two measurements. Is the answer correct? Use a labeled sketch, numbers, and words to explain.
Here is the formula for finding the area of a parallelogram.

- \textit{The area of a parallelogram} = \textit{base} \times \textit{height} or \textit{b} \times \textit{h} or \textit{bh}. (Since a rectangle is a special kind of parallelogram, this is also the formula for the area of a rectangle.)

3. Find the area of each figure below. Use the formulas. Show your work.

\begin{itemize}
  \item \textbf{a} \hspace{2cm} 6 \text{ cm} \\
  \hspace{2cm} 4 \text{ cm} \\
  \text{Area} = \text{_________ sq cm}
  \\
  \textbf{b} \\
  \hspace{2cm} 4 \text{ cm} \\
  \hspace{2cm} 6 \text{ cm} \\
  \text{Area} = \text{_________ sq cm}
\end{itemize}

4. For each of the parallelograms below:
  \begin{itemize}
    \item draw in the height.
    \item measure and label the height and the base to the nearest centimeter.
    \item find and record the area and show your work.
  \end{itemize}

\begin{itemize}
  \item \textbf{a} \\
  \hspace{2cm} \text{Area} = \text{_________ sq cm}
  \\
  \textbf{b} \\
  \hspace{2cm} \text{Area} = \text{_________ sq cm}
\end{itemize}
Classifying Quadrilaterals

A quadrilateral is any polygon that has 4 sides. There are many kinds of quadrilaterals, including:

- **trapezoid**: a quadrilateral with exactly 1 pair of parallel sides
- **parallelogram**: a quadrilateral with 2 pairs of parallel sides opposite each other
- **rectangle**: a parallelogram with 4 right angles
- **rhombus**: a parallelogram with 4 congruent sides
- **square**: a parallelogram with 4 congruent sides and 4 right angles

1. Look carefully at the figures below. Find out how many right angles, pairs of parallel sides, and pairs of congruent sides each has. Then circle all the words that describe the figure.

<table>
<thead>
<tr>
<th>Figure</th>
<th>How many right angles?</th>
<th>How many pairs of congruent sides?</th>
<th>How many pairs of parallel sides?</th>
<th>Circle the word(s) that describe(s) the figure.</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td></td>
<td></td>
<td></td>
<td>trapezoid parallelogram rectangle rhombus square</td>
</tr>
</tbody>
</table>
### Independent Worksheet 3  Classifying Quadrilaterals (cont.)

<table>
<thead>
<tr>
<th>Figure</th>
<th>How many right angles?</th>
<th>How many pairs of congruent sides?</th>
<th>How many pairs of parallel sides?</th>
<th>Circle the word(s) that describe(s) the figure.</th>
</tr>
</thead>
<tbody>
<tr>
<td>b</td>
<td></td>
<td></td>
<td></td>
<td>trapezoid, parallelogram, rectangle, rhombus, square</td>
</tr>
<tr>
<td>c</td>
<td></td>
<td></td>
<td></td>
<td>trapezoid, parallelogram, rectangle, rhombus, square</td>
</tr>
<tr>
<td>d</td>
<td></td>
<td></td>
<td></td>
<td>trapezoid, parallelogram, rectangle, rhombus, square</td>
</tr>
<tr>
<td>e</td>
<td></td>
<td></td>
<td></td>
<td>trapezoid, parallelogram, rectangle, rhombus, square</td>
</tr>
</tbody>
</table>

(Continued on next page.)
Independent Worksheet 3  Classifying Quadrilaterals (cont.)

Use a ruler marked in inches and the grid lines below to draw the following figures.

2 A rectangle with 4 congruent sides that are each $1\frac{1}{8}$ inches long

![Rectangle](grid.png)

3 A parallelogram with two sides that are each $2\frac{2}{8}$ inches long

![Parallelogram](grid.png)
**Set C1 ★ Activity 4**

**Three Mathematical Ideas**

**Overview**
Students investigate three mathematical ideas during this activity to generate and apply the formula for finding the area of a triangle.

**Skills & Concepts**
- ★ determine the formula for the area of a triangle by relating it to the area of a parallelogram
- ★ use formulas to determine the perimeters and areas of triangles and parallelograms

**You'll need**
- ★ Three Mathematical Ideas (page C1.40, run a copy on a transparency)
- ★ Finding the Area of Triangles (pages C1.41 and C1.42, run a class set)
- ★ paper to mask parts of the overhead
- ★ overhead pens
- ★ class set of geoboards and rubber bands
- ★ class set of rulers

**Instructions for Three Mathematical Ideas**

1. Let students know that you are going to spend some more time investigating perimeter and area today. Then place the top portion of the Three Ideas overhead on display, keeping the rest masked for now. Read the first statement with students, and ask them to take a minute to consider it privately. Do they agree? Why or why not?

<table>
<thead>
<tr>
<th>Three Mathematical Ideas</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 The formula for the area of any parallelogram, including a rectangle, is $\text{base} \times \text{height}$ or $bh$</td>
</tr>
</tbody>
</table>

2. Ask students to pair-share their thoughts for a minute or two. Then invite volunteers to share their ideas with the class.

   **Students** The formula for the area of a rectangle is length times width, not base times height.

   If you have a parallelogram that's slanted over, you have to find the height, and then multiply it by the base. You don't have to do that with a regular rectangle. You just multiply the two sides.

   I think you need two different ways to find the area, one for rectangles, and one for the other parallelograms.

   I agree. I don't think they can be the same.

3. While some students may believe that the formula for the area of a rectangle is distinctly different from the formula for the area of a non-rectangular parallelogram, a few might challenge this assumption by pointing out that the height of a parallelogram is the same as the width of a rectangle. If students don't raise this issue, press their thinking by reviewing the definition of height with the class.

   **Teacher** Let's think some more about this idea together. What is height? Who can tell us what that term means? Pair-share your ideas, and then let's hear from some of you.
**Activity 4 Three Mathematical Ideas (cont.)**

*Students*  It's how high the shape is, like how far up it goes.
It's how far it is from one side to the other, but it's up and down instead of sideways.
It has to make a right angle with the side on the bottom—the base.
It's kind of like the width of a rectangle, but you have to draw it in.

4. Reveal the figures below the first statement, and ask volunteers to show the height of each.

*Teacher*  Jessa and Carlos showed us the height of the two parallelograms. What about the height of the rectangle? Can anyone show us that?

*Lani*  You don't have to. It's already there. You don't have to draw anything!

*Teacher*  I thought you all agreed that the height of a figure tells how far one side is from its opposite side, and that the height has to be perpendicular to the base. Let's look more closely at the width of the rectangle. Doesn't it tell how far it is from one side of the rectangle to its opposite side?

*Austin*  Yes, and it's also perpendicular to the base! You don't have to draw it, but it's the same as the height on the other parallelograms!

5. Even though you may not have total agreement, ask students to give the formula a road test. Work with their input to determine the base and height of each figure, multiply the two dimensions, and record the area. As you do so, let them know that the grid is marked off in centimeters. Then ask volunteers to verify the answers visually. Are they all correct?
Students You can see that if you slide the triangle over on the bottom parallelogram, it’s going to make a 3-by-3 square. That’s 9 square centimeters. On that other parallelogram, if you think about cutting off the triangle and moving it over, you’ll get a 3-by-5 rectangle. That’s 15 square centimeters. They’re all right, because three rows of seven is 21, so the rectangle is 21 square centimeters.

6. Now reveal the second mathematical idea on the overhead and read it with the class. Do students agree with this statement? Why or why not? Give them a minute to consider it privately. Then have them pair-share their thinking, and call on a few volunteers to share their ideas with the class.

2 Any triangle is half of a parallelogram.

Students That seems right. You can put two triangles together to make a rectangle or a square. I bet I could make a triangle that wouldn’t work. I think you can split any parallelogram into 2 triangles. I think it’s right.

7. Show the illustrations below the second statement.

Ask students to imagine that each of the triangles is only half of a larger figure. Could that larger figure be a parallelogram? Give students each a geoboard and some rubber bands, and ask them to test the idea for themselves. Suggest they work in pairs, that each partner create one copy of the same triangle, and they superimpose one of their boards on top of the other to make a parallelogram. Ask them to experiment with all three of the triangles shown on the overhead, and if they have extra time, to test the idea with other triangles on their geoboards.

8. After a few minutes, invite volunteers up to share their results by placing their geoboards on the overhead or sketching on the transparency. Did each triangle turn out to be half of a parallelogram? Did anyone find a triangle that didn't appear to be half of a parallelogram? Are they convinced that the statement is true?

9. Now display the third idea. Read it with the class, and clarify as needed. Does it make sense? Will it work?
10. Erase any marks that may have been made on the lower half of the transparency so only the triangles remain. Work with input from the class to determine the base and height of the first triangle. Then have students multiply the two dimensions and divide the result in half to find the area of the triangle as you record on the transparency. Repeat this with the second and third triangles.

11. Then ask volunteers to build each of the triangles on their geoboard and verify the answers. Is the area of the first triangle actually 3? What about the second triangle? Can they convince one another that the area of the third triangle is 1 1/2? Ask volunteers to bring their geoboards to the overhead to demonstrate that the areas are correct.

**Darius** I made the first triangle on my board. Then I made a rectangle around it. You can see that the area of the rectangle is 6 squares, so the triangle has to be 3. The formula worked on that one.

**Rosa** I put the second triangle on my board and made 2 rectangles to help figure out the area. You can see that the little part of the triangle is worth 1, and the bigger part is worth 2. That’s 3 in all, so the formula gave us the right answer.

**Sam** I did the same thing as Rosa on the third triangle. It came out to be half a square on the left part of the triangle, and half a rectangle of 2 on the right hand triangle. That’s a half and one. The formula said the area was 1 1/2, so it worked.

12. Give students each a copy of Finding the Area of Triangles. Review the instructions on both sheets with the class. When students understand what to do, let them get started. Plan to assign unfinished work as homework or seatwork the following day.
Finding the Area of Triangles

The formula for finding the area of all parallelograms, including rectangles is base \times height, or \(bh\).

1. Use the formula to find the area of the two parallelograms below.

   \[
   \text{Area} = \quad \text{sq cm}
   \]

2. Find the area of Triangle ABC and Triangle DEF. Use numbers, words, and labels on the sketches to explain your answers.

   \[
   \text{Area} = \quad \text{sq cm}
   \]

3. The formula for the area of a triangle is \(\frac{1}{2}\) base \times height, or \(\frac{1}{2}bh\). Use labeled sketches, numbers, and words to explain why this works.

4. For each of the triangles below:
   - draw in the height.
   - measure and label the height and the base to the nearest centimeter.
   - find and record the area.
   - show your work.

   \[
   \text{Area} = \quad \text{sq cm}
   \]

INDEPENDENT WORKSHEET

Use Set C1 Independent Worksheets 5 & 6 to provide students with more practice using formulas to determine the perimeters and areas of triangles and parallelograms.
Three Mathematical Ideas

1 The formula for the area of any parallelogram, including a rectangle, is

\[ \text{base} \times \text{height or } bh \]

2 Any triangle is half of a parallelogram.

3 If any triangle is half of a parallelogram, and the area of any parallelogram is \( \text{base} \times \text{height} \), the formula for the area of any triangle must be \( \frac{1}{2} \text{ base} \times \text{height} \), or \( \frac{1}{2} bh \).
The formula for finding the area of all parallelograms, including rectangles is \( \text{base} \times \text{height}, \) or \( bh \).

1 Use the formula to find the area of the two parallelograms below.

\[
\begin{align*}
\text{a} & : \quad 6 \text{ cm} \\
\text{4 cm} & \\
\text{Area} = \underline{\quad} \text{ sq cm} \\
\text{b} & : \quad 4 \text{ cm} \\
\text{6 cm} & \\
\text{Area} = \underline{\quad} \text{ sq cm}
\end{align*}
\]

2 Find the area of Triangle ABC and Triangle DEF. Use numbers, words, and labels on the sketches to explain your answers.

\[
\begin{align*}
\text{a} & : \quad 6 \text{ cm} \\
\text{4 cm} & \\
\text{A} & \text{B} \quad \text{C} \\
\text{Area} = \underline{\quad} \text{ sq cm} \\
\text{b} & : \quad 4 \text{ cm} \\
\text{6 cm} & \\
\text{D} & \text{E} \quad \text{F} \\
\text{Area} = \underline{\quad} \text{ sq cm}
\end{align*}
\]
3 The formula for the area of a triangle is \( \frac{1}{2} \text{base} \times \text{height} \), or \( \frac{1}{2} bh \). Use labeled sketches, numbers, and words to explain why this works.

4 For each of the triangles below:
   - draw in the height.
   - measure and label the height and the base to the nearest centimeter.
   - find and record the area.
   - show your work.

\begin{align*}
\text{Area} &= \quad \text{sq cm} \\
\text{Area} &= \quad \text{sq cm}
\end{align*}
Quad Construction

A quadrilateral is any polygon that has 4 sides. There are many kinds of quadrilaterals, including:

1. **Trapezoid**
   - A quadrilateral with exactly 1 pair of parallel sides.

2. **Parallelogram**
   - A quadrilateral with 2 pairs of parallel sides opposite each other.

3. **Rectangle**
   - A parallelogram with 4 right angles.

4. **Rhombus**
   - A parallelogram with 4 congruent sides.

5. **Square**
   - A parallelogram with 4 congruent sides and 4 right angles.

Use a ruler marked in inches and the grid lines below to draw the following figures.

1. A trapezoid with one right angle, one side length of 17⁄8 inches and one side length of 25⁄8 inches.

2. A parallelogram that is not a rectangle with an area of 18 square units. (The smallest square on the grid has an area of 1 square unit.) Label your drawing to prove that the area is 18 square units.

(Continued on back.)
3 A parallelogram with 4 right angles and an area of 32 square units. Label your drawing to prove that the area is 32 square units.

4 A parallelogram that is not a rectangle with an area of 32 square units. Label your drawing to prove that the area is 32 square units.

5 Darius says it is impossible for a trapezoid to have parallel sides that are also the same length. Remember that a trapezoid is any quadrilateral with exactly 1 pair of parallel sides. Explain why you agree or disagree with Darius. Draw on the grid to help explain your answer.
Perimeter & Area Puzzles

To find the perimeter of any triangle or quadrilateral, add the side lengths. For rectangles, you can use the formula \(2l + 2w\).

The formula for finding the area of all parallelograms, including rectangles is base \(\times\) height, or \(bh\).

The formula for finding the area of all triangles is \(\frac{1}{2}\) base \(\times\) height, or \(\frac{1}{2} bh\).

1 Use the formulas above to find the perimeter and area of each figure on this page. Show your work.

<table>
<thead>
<tr>
<th>Figure</th>
<th>Perimeter</th>
<th>Area</th>
</tr>
</thead>
<tbody>
<tr>
<td>a Square</td>
<td></td>
<td></td>
</tr>
<tr>
<td>b Parallelogram</td>
<td></td>
<td></td>
</tr>
<tr>
<td>c Equilateral Triangle</td>
<td></td>
<td></td>
</tr>
<tr>
<td>d Rectangle</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(Continued on back.)
2 Fill in the blanks to label each of the shapes below with its dimensions, perimeter, and/or area. Use the information in each drawing to help. Show your work.

**Example** Rectangle

- Perimeter = 24 feet
- Area = \(5 \times 7 = 35\) square feet

<table>
<thead>
<tr>
<th>5 ft.</th>
<th>7 ft.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>5 ft.</th>
<th>7 ft.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**a** Square

- Perimeter = 16 feet
- Area = _____ square feet

<table>
<thead>
<tr>
<th>4 ft.</th>
<th>_____ ft.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>4 ft.</th>
<th>_____ ft.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**b** Parallelogram

- Perimeter = _____ feet
- Area = _____ square feet

<table>
<thead>
<tr>
<th>8 ft.</th>
<th>4 ft.</th>
<th>h = 3 ft.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>8 ft.</th>
<th>4 ft.</th>
<th>h = 3 ft.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**c** Equilateral Triangle

- Perimeter = _____ feet
- Area = _____ square feet

<table>
<thead>
<tr>
<th>6 ft.</th>
<th>h = 4 ft.</th>
<th>_____ ft.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>6 ft.</th>
<th>h = 4 ft.</th>
<th>_____ ft.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**d** Right Triangle

- Perimeter = _____ feet
- Area = _____ square feet

<table>
<thead>
<tr>
<th>3 ft.</th>
<th>4 ft.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>3 ft.</th>
<th>4 ft.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**e** Parallelogram

- Perimeter = _____ feet
- Area = _____ square feet

<table>
<thead>
<tr>
<th>5 ft.</th>
<th>4 ft.</th>
<th>h = 3 ft.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>5 ft.</th>
<th>4 ft.</th>
<th>h = 3 ft.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Ebony and her mom are going to make a quilt for Ebony's bed. When it is finished, the quilt will be 72 inches by 90 inches.

1. How many square inches will Ebony's quilt be in all? Show your work.

2. Ebony and her mom went to the store to buy fabric for the quilt. They picked out 4 colors they liked. The fabric was 45 inches wide. Ebony said, “Let's buy 4 feet of each color.”

Will that be enough fabric to make the quilt? Show all your work.

Here is one of the 4 pieces of fabric they bought.

(Continued on back.)
3 Ebony's quilt will be made out of squares that are 9 inches on each side. Make a labeled sketch on the grid below to show how Ebony and her mom will have to arrange the squares to make a 72" by 90" quilt.

4 Ebony is planning how she wants each 9-inch square to look. Here is her first plan. What is the area of the light grey triangle? Show your work.

The area of the light grey triangle is ________ square inches.

5 Her brother Noah said, "Why don't you use all 4 colors in each quilt square? Here's a different plan." What is the area of the light grey triangle in Noah's plan? Show your work.

The area of the light grey triangle is ________ square inches.
Set D2 ★ Activity 1

Introducing Volume

Overview
In this activity, students move toward increasingly efficient methods of finding the volume of cubes and rectangular solids.

Skills & Concepts
★ determine volume by finding the total number of same-sized units of volume that fill a three-dimensional shape without gaps or overlaps
★ understand a cube that is one unit on an edge is the standard unit for measuring volume
★ select appropriate units, strategies, and tools for solving problems that involve estimating or measuring volume
★ measure necessary attributes of shapes to use volume formulas to solve problems

You’ll need
★ Cubes & Rectangular Solids (page D2.4, run 1 for display)
★ Paper Box Pattern (page D2.5, run a class set)
★ centimeter cubes (class set)
★ scissors
★ scotch tape
★ rulers (class set)
★ Student Math Journals
★ Word Resource Cards (congruent, edge, face, parallel, perpendicular, vertex) optional

Advance Preparation Display the Word Resource Cards where students can see them before conducting the activity.

Instructions for Introducing Volume
1. Give students each a centimeter cube and allow several minutes for them to record as many observations as they can about the cube in their math journals. Call their attention to the Word Resource Cards before they start writing and challenge them to include at least 3 of the words in their observations.

2. Have them pair-share their observations, and then call for whole-group sharing. Record some of their observations at the top of the Cubes and Rectangular Solids, keeping the rest of the page covered for now. If it doesn't come up in the discussion, ask students to find examples of parallel, perpendicular, and congruent edges and faces as they examine their cubes.

3. Ask students to estimate the length of one of the edges of their cube. Then have a volunteer measure to confirm that each edge is 1 centimeter. Next, ask students to determine the area of one of the cube's faces. Finally, explain that because their cube is 1 centimeter long, wide, and high, it is called a cubic centimeter. Just as centimeters are used to measure length and square centimeters are used to measure area, cubic centimeters are used to measure volume. Add this information to the display, along with the abbreviations for each measure.
4. Next, reveal the picture of the rectangular solid on the display. Have students write at least 3 observations about this figure in their journals. Then invite volunteers to share their observations with the class as you record at the display. After you’ve recorded 8–10 observations, work with input from the students to label all 3 dimensions of the solid: length, width, and height.

5. Give each table a good supply of centimeter cubes. Ask each student to build several different rectangular solids that have a volume of exactly 12 cubic centimeters. Be sure they understand that their constructions have to be solidly filled in, without gaps or holes between cubes. Ask them to share and compare their constructions as they’re working.

6. After a few minutes, call a halt to the construction process. Ask several volunteers to describe their constructions by length, width, and height. Record each description at the board, along with an equation to confirm that the total is 12 cubic centimeters.

7. Now reveal the rectangular solid at the bottom of the display. Ask students to replicate it with their cubes and determine its volume without counting every cube one by one. As they finish, invite volunteers to share their strategies with the class, as you record at the display. If it doesn't come from one of the students, ask them what would happen if you multiplied length × width × height. Would it result in the same answer they’ve shared? Why or why not? Press them to explain their thinking and then work with their input to write the equation and solve the multiplication problem.
8. Ask students to clear their cubes to the side for now and get out their scissors. Give each student a copy of the Paper Box Pattern and supply each table with some scotch tape. Have them cut, fold, and tape their paper patterns to make a box. Ask early finishers to help others near them.

9. When everyone has finished, ask students to estimate the volume of the box. How many centimeter cubes do they think it will take to fill the box completely? Record some of their estimates on the board. Then challenge them to work in pairs to determine the actual volume of the box without filling it to the top with cubes, dumping them out, and counting them one by one. As they finish, have them record their solution in their journal, along with a detailed description of their strategy.

10. Toward the end of the period, reconvene the class. Ask volunteers to share their strategies and solutions with the class. If the idea of measuring the dimensions of the box and multiplying them doesn't come from one of the students, ask them to get out their rulers and try it. Does it result in the same solution they got using other methods? Why? (Students should find that the taped box holds 54 centimeter cubes. It is 6 centimeters long, 3 centimeters wide, and 3 centimeters high. $6 \times 3 \times 3 = 54 \text{ cm}^3$.)
Cubes & Rectangular Solids

Cube

Rectangular Solid

Build this rectangular solid with your centimeter cubes. Find the volume *without* counting each cube 1 by 1.
Paper Box Pattern

Cut out this pattern. Fold along the dashed lines and tape to make a box.
Set D2 ★ Activity 2

More Paper Boxes

Overview
Using paper boxes and centimeter cubes, students work together to generate efficient methods, including the standard formulas, for finding the volume of cubes and rectangular solids.

Skills & Concepts
★ determine volume by finding the total number of same-sized units of volume that fill a three-dimensional shape without gaps or overlaps
★ select appropriate units, strategies, and tools for solving problems that involve estimating or measuring volume
★ measure necessary attributes of shapes to use volume formulas to solve problems

You’ll need
★ More Paper Boxes (page D2.10, run a half-class set, plus a few extra)
★ Student Math Journals or Journal Grid Page (page D2.11, run as needed, optional)
★ centimeter cubes (class set)
★ scissors
★ scotch tape
★ rulers (class set)
★ Counting on Frank by Rod Clement (optional)

Instructions for More Paper Boxes
1. Ask students to pair up, or assign partners. Students will need their rulers, scissors, and journals for this activity. Give each pair a copy of More Paper Boxes, along with some scotch tape. Have them cut out and tape together Box A, leaving Boxes B and C uncut for now. Ask early finishers to help others nearby.

2. When most students have finished constructing Box A, ask them to estimate how many centimeter cubes it will take to fill the box completely. Have them each record an estimate in their journals. Then ask volunteers to share and explain their estimates as you record at the board.

Lauren It looks like it’s going to take about 10 to fill the bottom, and it’s about 3 cubes high, so I think 30 cubes will do it.

Tonio I say 40 because it’s maybe 10 on the bottom and 4 up. That would be $4 \times 10$, so that’s 40.

Marisa I said 54 cubes because it looks like it’s 3 across and maybe 6 long. That’s 18. I think it’s going to be 3 layers high, so I multiplied $3 \times 18$ to get 54.

3. Distribute centimeter cubes and ask student pairs to find the actual volume of Box A. Encourage students to use any efficient method they choose, but moving away from filling the box completely, dumping out the cubes and counting by ones. As they finish, have them record their answer, along with a description of their strategy in their journal.

4. After they’ve had a few minutes to work, ask volunteers to share their solutions and strategies with the class.
5. If the idea of measuring the dimensions of the box and multiplying them doesn’t come from the students, ask them to get out their rulers and try it. Does this strategy result in the same solution they got using other methods? Why? Work with class input to record an equation that matches what they just did: $7 \times 3 \times 4 = 84 \text{ cm}^3$.

6. Ask students to cut out and tape together Box B and record an estimate of the volume in their journals. As they’re working, collect the centimeter cubes. When most have finished, ask volunteers to share their estimates as you record at the board. Then challenge students to find the actual volume of the box using their rulers instead of cubes. Have them record the answer, along with any computations they made, in their journal.

7. After they’ve had some time to work, ask volunteers to share their solutions and strategies with the class. Then work with input from the class to write a general formula for finding the volume of a rectangular solid ($\text{length} \times \text{width} \times \text{height} = \text{volume}$), along with an equation for Box B ($6 \times 4 \times 2 = 48 \text{ cm}^3$). Have students record this information in their journals.

8. Now tell them that some fifth graders in another class said they thought they could find the volume of Box C without cutting and taping it together. Do your students agree with these fifth graders? Why or why not? Have them pair-share their responses and then ask volunteers to share their thinking with the class.

Students We said you could do it by just using a ruler, but you should cut out the box and put it together first.

We think they’re right. It looks like it’s going to be a cube, so if you just measured one edge, you could figure it out.

9. Ask students to measure one or more edges of the uncut box to help make as accurate an estimate as possible. Have them record their estimate, along with an explanation in their journal. (If they’re sure their estimate matches the actual volume, that’s fine.)

10. After a few volunteers have shared and explained their estimates, ask students to cut out and tape together Box C. Have them measure it to determine the actual volume, and record the answer, along with any calculations they made, in their journals.

11. Have volunteers share and explain their solutions and strategies. Was it possible to determine the volume of the figure by measuring only 1 edge? Why or why not? Would it have been possible to find the answer without cutting and taping the cube? Why or why not? Then have students write an equation for the volume of Box C ($4 \times 4 \times 4 = 64 \text{ cm}^3$) in their journals.
Activity 2  More Paper Boxes (cont.)

Extensions
• Explain that because the length, width, and height of a cube are all equal, mathematicians generally represent \( s \times s \times s \) by \( s^3 \). Using this notation, the volume of a cube is \( s^3 \) where \( s \) is the length of one edge of the cube.

\[
s \times s \times s = s^3
\]

Record this at the board, and ask students to compare it to the formula for finding the volume of a rectangular prism. How are the two alike? How are they different? Ask them to record the general formula for finding the volume of a cube in their journals.
• Have volunteers use lightweight cardboard and tape to construct a cubic inch and a cubic foot, and share them with the class. Ask students to list in their journals some of the things they’d measure in cubic inches and some of the things they’d measure in cubic feet.
• Read Counting on Frank by Rod Clement before or after this session.

INDEPENDENT WORKSHEET

See Set D2 Independent Worksheet 1, Volume Review (page D2.35) and Independent Worksheet 2, The Camping Trip (page D2.39) for more practice selecting and using appropriate units and formulas to determine length, area, and volume.
More Paper Boxes

A

B

C
Journal Page Grid
Volume Review

*Volume* is the measure of the space occupied by a 3-dimensional object. Volume is measured in cubes of a given size, such as cubic centimeters, cubic inches and cubic feet.

1. Each of the rectangular solids below was built with centimeter cubes. Label each with its dimensions (length, width, and height) and find the volume. Show your work.

**example**

\[
\text{Volume } 4 \times 2 \times 3 = 24 \text{ cubic cm (or cm}^3)\]

**a**

\[
\text{Volume }
\]

**b**

\[
\text{Volume }
\]

**c**

\[
\text{Volume }
\]

(Continued on next page.)
Independent Worksheet 1  Volume Review (cont.)

2 Use the centimeter side of your ruler to measure the dimensions of each rectangular solid below. Then find its volume. Show your work.

**Example**

Volume \(4 \times 3 \times 2 = 24 \text{ cubic cm (or cm}^3)\)

**a**

**Volume**

**b**

Volume

(Continued on next page.)
3 Miguel says you only need to measure one edge of a cube to find its volume. Do you agree with him? Why or why not? Use numbers, labeled sketches, and words to explain your answer.

4 Mia has already measured the dimensions of this packing box. Help her find the volume. Show your work.

5 Brandon is going on a fishing trip with his family. He wants to find the volume of the family’s ice chest. Which expression should he use?

- \(2 \times 3\)
- \(3 \times 2 \times 2\)
- \(3 + 2 + 2\)
- \((3 \times 2) - 2\)

6 Jeff’s little brother is trying to find out how many alphabet blocks will fit into a shoebox. He is measuring:

- the volume of the shoebox
- the area of the shoebox
- the length of the shoebox

(Continued on next page.)
7 Which of these situations is about volume?

- determining the amount of fencing it takes to go around a square garden
- determining how many tiles it will take to cover the kitchen floor
- determining how many rectangular containers of food will fit into a freezer

8 Vanesa wants to find the volume of her lunchbox. Which of these units should she use?

- cubic feet
- cubic inches
- cubic yards

9 The volume of this rectangular solid is 40 cubic feet. What is its height? Show your work.

10 The volume of this cube is 125 cubic inches. What is the length of each edge? Show your work.
The Camping Trip

The Gomez family is going on a camping trip next week. There are 4 people in the family: Mr. and Mrs. Gomez and the 11-year-old twins, Ramon and Dora. Help them do some planning for their trip. Circle a correct answer to each question below.

1 Mrs. Gomez wants to cut a piece of rope that's long enough to dry the family's laundry on every day. Which of these units should she use to measure the rope?
   - inches
   - feet
   - miles

2 Mr. Gomez wants to figure out how far they’ll have to drive to get to the campsite. He already knows that it will take about a day to get there. Which of these units should he use?
   - inches
   - feet
   - yards
   - miles

3 The shoelaces on Ramon's tennis shoes are almost worn out. He has to measure them so he gets the right length at the store. Which of these units should he use?
   - millimeters
   - centimeters
   - meters
   - kilometers

4 Mrs. Gomez says it's going to be a 3-minute walk from their tent to the lake. Dora wants to measure the distance when they get there. Which of these units should she use?
   - millimeters
   - centimeters
   - meters
   - kilometers

5 Ramon wants to find the area of his sleeping bag to see how much room he'll have in the family's tent. Which of these units should he use?
   - square inches
   - square feet
   - square miles

(Continued on next page.)
6 Which formula should Ramon use to find the area of his sleeping bag?

Area = Length + Width       Area = Length × Width       Area = Length ÷ Width

7 Dora says when they get there, she's going to measure the area of their campsite. Mrs. Gomez says the campsite is big enough for their car, their tent, their picnic table and chairs, and their campfire, with a little room left over. Which of these units should she use?

- square inches
- square yards
- square miles

8 Which formula should Dora use to find the area of the campsite?

A = (2 × l) + (2 × w)       A = (3 × l) − (2 × w)       A = l × w

9 Mr. Gomez wants to find the volume of the family car trunk so he'll know how much luggage will fit back there. Which of these units should he use?

- cubic inches
- cubic feet
- cubic yards

10 Ramon wants to measure the volume of a shoebox to find out how many CD's he can fit into it for the trip. Which of these units should he use?

- cubic inches
- cubic feet
- cubic yards

11 Dora is going to collect tiny pebbles at the lake. She wants to measure the volume of a metal band-aid box to keep them in. Which of these units should she use?

- cubic centimeters
- cubic meters
- cubic kilometers
Fish Tank Volume

Overview
During this activity, students will demonstrate their understanding of the properties of solid figures and determine the volume of rectangular prisms. They convert measurement units within a given measurement system, and use these conversions in solving multi-step, real world problems with volume.

Skills & Concepts
- Recognize volume as an attribute of solid figures and understand concepts of volume measurement.
- Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units.
- Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base.
- Represent threefold whole-number products as volumes, the associative property of multiplication.
- Apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole number edge lengths in the context of solving real world and mathematical problems.

Instructions for Fish Tank Volume
1. Begin by telling students that over the next few days they will build several models of fish tanks that meet specific volume requirements. Activate their prior knowledge about fish tanks by asking the following questions: How many students have a fish tank at home? What do you know about fish tanks? Where could a fish tank go in the office area?

2. Read the Design a Fish Tank scenario. Discuss the properties of a cube and invite one student to draw a cube in the display area, while others do the same in their journals. (Some students may need additional support when drawing a cube but this is not a critical skill and a quick sketch will do). Restate the total fish tank volume would be 8 cubic feet of water.

Teacher What does the word “cubic” mean?
Sarah  A cube is a three-dimensional figure where all sides are equal.

Jacob  You show the total volume by adding the little 3 behind whatever unit you are measuring in. The little 3 means that the shape is 3-D.

Teacher  How do we find the area of a cube?

Students  Length times width times height.

Teacher  So if the tank has a total volume of 8 cubic feet how can we find the length, width, and height of the tank?

Kristen  I think we should take the 8 and divide it by 3 because there are three types of sides.

Betsy  Wait! If we divided 8 by 3 we would get an answer of $2\frac{2}{3}$ Now if I use the formula of length x width x height and plug in $2\frac{2}{3}$ that number would be greater than 8 cubic feet.

Teacher  We know that we need all the sides to be the same in length when our tank is in the shape of a cube. What is one way to represent the quantity, 8?

Betsy  I know if the cube had the dimension of $2 \times 2 \times 2 = 8 \text{ ft.}^3$

3. Distribute handfuls of centimeter cubes to each group and ask each student to count out 8 cubes. Have them build the cube model they have drawn and then label the dimensions in their journals. Now ask the students to manipulate the same 8 cubes in a variety of ways to create different rectangular prisms. Ask students to demonstrate their ideas as a class, while you record a quick sketch and the dimensions on the teacher master. Have students record the same information in their journals.

4. Give students a few moments to consider the next question: How many cubic inches are in 8 cubic feet? Have students turn and share their thinking and then record a solution in their journal. Calculators might be helpful. Solicit strategies from the class and discuss how the students computed the answer 13,824 in.3.

5. Set the stage with some background information. People often buy a fish tank that is too small for even one fish. Encourage students to estimate about how much space a fish would need to survive. A general ratio is one fish per one gallon of water. One gallon of water is approximately cubic feet of water.

6. Display question 2 of the Design a Fish Tank master. As a class, determine how much water is needed for a fish to survive in feet and then in inches, and record the solutions.

7. Ask students to discuss with a partner how many fish could live comfortably in 1 cubic foot of water.

8. Move on to Fish Tank Models. Display a copy, read the scenario and specifications at the top. Have students brainstorm, first in pairs and then as a whole group, what are some possible dimensions of fish tanks that will work with these specifications. Using cubes, grid paper or quick sketches, have the students begin their inquiry.

9. Encourage students to record the models in their journals, including labels for the dimensions. Once 3 different models have been built, select three pairs of students to share their thinking with the class.

10. As you discuss the different tank volumes, pose the following questions:
    • What might be a more efficient modeling strategy than using centimeter cubes?
Activity 3 Fish Tank Volume (cont.)

- What formula might apply in this situation?
- What is an equation or expression that matches these models?
- What would happen if we changed the order of the numbers?

**Teacher**  What might be a more efficient modeling strategy than using centimeter cubes?

**Sammy**  We could use boxes that are in the shapes of cubes or draw a picture to represent the units—a 3-D box with labels.

**Teacher**  Good, pictures could be a definite solution.

**Reece**  Well we have a length, a width, and a height. Could we use those to make the formula of Length x Width x Height?

**Abby**  Yes Reece, we could even shorten it to L x W x H.

**Teacher**  Which letter is which on the model?

**Abby**  The Length is the bottom measurement facing us. The width is how deep the object is. That's the measurement going away from us. And the height is what measurement is pointing towards the top of the paper.

**Teacher**  What is the equation or expression that matches these models?

**Sarah**  I think we just need to plug in the numbers we are using into the formula to create an equation that represents our model.

**Teacher**  Great work everyone! Now what would happen if we changed the position and/or order of the numbers? Would we get the same answer?

**Reece**  This is multiplication formula so the order doesn’t matter. We would get the same number any way we change the order of the numbers around.

**Abby**  I agree with Reece. I think it’s the commutative property.

11. Continue to discuss solution strategies for the three questions below the table as a class.

**Extension**

Have students measure out the length and width of their model in feet with masking tape on the floor to build a correlation between the model and the actual size of the tank.

Use Independent Worksheet 3, Tank Volumes (page D2.41) for additional practice with volume.
Design a Fish Tank

Your school wants to install several fish tanks in the office area. Your class has been asked to design the fish tanks and choose which fish to buy.

1 One tank the office considered purchasing held a total of 8 cubic feet of water.
   a What would be the dimensions of the tank be if the shape was a cube?

   b What other dimensions could the tank be if it didn't have to be a cube?

   c How many cubic inches would that tank hold?

2 The table that follows represents the dimensions and area of a fish tank for just one fish (without rocks and plants). If each fish needs 1 cubic unit with the dimensions of \( \frac{1}{2} \) of a foot for the length, width and height of water to live in, what is the total volume in feet? Inches?

<table>
<thead>
<tr>
<th>Number of Fish</th>
<th>Volume of Water</th>
<th>Total Volume</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} )</td>
<td>[ \text{ft.}^3 ]</td>
</tr>
<tr>
<td>1</td>
<td>[ \text{______} \times \text{______} \times \text{______} ]</td>
<td>[ \text{in.}^3 ]</td>
</tr>
</tbody>
</table>

3 If one small fish needs \( \frac{1}{8} \) cubic feet of water to live in, what is the maximum number of small fish that would fit in 1 cubic foot of water? How do you know?
Fish Tank Models

1 Your classmates will be presenting several fish tank models to the office staff. The staff will vote on which tank sizes would work the best. Your job is to create 3 models that meet these specifications:
• The volume of the actual fish tank can be no larger than 16 cubic feet.
• The volume of the actual fish tank can be no smaller than 4 cubic feet.
• The minimum base size of the tank can be no smaller than 2 square feet.

For your design, each cm cube is equal to 1 foot cubed (1 cm$^3$ = 1 ft.$^3$)

<table>
<thead>
<tr>
<th>Tank</th>
<th>Length (in cm)</th>
<th>Width (in cm)</th>
<th>Height (in cm)</th>
<th>Volume of the model (in cm$^3$)</th>
<th>Actual volume of the tank (in ft.$^3$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2 Answer the following questions based on the information above.

a Which tank model has the largest base? ___________
Explain.

b Which tank model has the smallest base? ___________
Explain.

c If the office selects the largest tank volume, how many fish can that tank hold? (Remember each fish needs $\frac{1}{8}$ cubic feet of water to live in).
Journal Page Grid
Tank Volume

1 At a pet store, the volume of a tank or kennel depends on the size of the pet. Use cubes, sketches and equations to solve for the following questions.

**a** What is the volume (including units) of the ant farm, if each cubic unit is one centimeter cubed?

[Diagram of ant farm: 10 cm x 2 cm x 8 cm]

Ant Farm Volume =

**b** What is the volume (including units) of the dog kennel, if each cubic unit is one foot cubed?

[Diagram of dog kennel: 6 ft x 7 ft x 7 ft]

Dog Kennel Volume =

**c** What is the volume (including units) of the scorpion tank, if each cubic unit is one inch cubed?

[Diagram of scorpion tank: 9 in x 7 in x 9 in]

Scorpion Tank Volume =

**c** What is the volume (including units) of the Guinea Pig cage, if each cubic unit is one inch cubed?

[Diagram of Guinea Pig cage: 16 in x 18 in x 30 in]

Guinea Pig cage Volume =

(Continued on next page.)
Pet Food Volumes

2 The pet store sells pet food and snacks in four sizes. Determine the volume of each container and write an equation to show your thinking.

a

\[
\text{Volume } = 9 \times 9 \times 9
\]

Equation

b

\[
\text{Volume } = 30 \times 19 \times 6
\]

Equation

c

\[
\text{Volume } = 9 \times 17 \times 3
\]

Equation

d

\[
\text{Volume } = 21 \times 16 \times 5
\]

Equation

3 Answer the following questions using the information above.

a Which figure has a volume less than the volume of figure A?

b What are the volumes of figure A and figure B combined?

e List the figures in order from least to greatest.
Set D2 ★ Activity 4

Number of Fish

Overview
The saga of the fish tanks in the office continues today, as students determine the number of fish that will live comfortably in a variety of different fish tanks. Students will select fish of different lengths and create a line plot to show their data. They will use this data to determine the mean, median, range and mode of their data set in fractional units. The second half of the session provides additional practices with a second scenario.

You'll need
★ Number of Fish (page D2.21, run 1 for display)
★ Length of Fish, Line Plot Data Analysis (page D2.22, run a class set, plus 1 for display)
★ Second Tank Needs Fish (page D2.23 and D2.24, run a class set, plus 1 for display)
★ Word Resource cards for range, median, mode, mean, line plot and x-axis (optional)
★ Colored Tiles (class set)

Skills & Concepts
★ Solve word problems involving addition and subtraction of fractions using visual fraction models or equations to represent the problem.
★ Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers.
★ Solve word problems involving division of whole numbers using visual fraction models or equations to represent the problem.
★ Solve real world problems involving multiplication of fractions and mixed numbers, using visual fraction models or equations to represent the problem.
★ Convert among different-sized standard measurement units within a given measurement system.
★ Make a line plot to display a data set of measurements in fractions of a unit (1⁄2, 1⁄4, 1⁄8).
★ Use operations on fractions for this grade to solve problems involving information presented in line plots.

Instructions for Number of Fish
1. Open today's session by displaying the Number of Fish master. Read the top portion together and invite the students to work with a partner to select 8 fish from the collection. Select 8 children to share their choices, ensuring that there are at least 3 different types of fish represented in the collection, then record the final choices on the teacher master.

2. Today, each of the colored tiles will represent 1⁄4 of a foot. Students will work in teams to build a model for all 8 of the fish lengths chosen by the class, representing the actual fish length in tile.
3. Reveal the Length of Fish, Line Plot Data Analysis page and work together to represent the length of 8 fish on a line plot. Title the line plot and label the x-axis the appropriate half, quarter or whole unit. You may need to review the definition of an x-axis.

4. Determine the median, range and mode of the fish lengths. To efficiently find a solution, the fish lengths should be displayed from least to greatest. Use the Word Resource cards to review the terms as needed.

5. Challenge students to find the total length of all 8 fish combined. Solicit several possible strategies and encourage each team to write an equation to match their computation.

6. Ask student pairs to find the average or mean length of the fish using the tile or a computation strategy. Invite several volunteers with varying ideas to share their work with the class.

   **Kale** We used the tile to make equal columns.

   **Sandra** I found the average by adding up all of the fourths and dividing by 4. My answer is \( \frac{16}{4} \) which equals 4 and then I divided by 4 to get 1 foot.

   **Jessy** You are partially correct Sandra. You do need to add all of the fraction but then you need to divide by how many fish we had, not the four lengths. I would have used the 4 and divided by 8 to get \( \frac{1}{2} \) of a foot. We had 8 fish.

7. Introduce the second scenario, Second Tank Needs Fish and follow the same teaching sequence. Allow students to choose to work alone, in pairs or with you. You may choose to reteach in a small group with the tile models while other students work more independently.

**Extension**

Challenge problems are provided.

**INDEPENDENT WORKSHEET**

Use Independent Worksheet 4, Third Tank Needs Fish (page D2.43) for additional practice with line plots.
Number of Fish

The school secretary decided that she wants 8 fish in the largest fish tank model you built yesterday. These fish are going to need more water than a normal fish due to their sizes. The secretary would like to see a minimum of 3 different types of fish in this tank. Work with a partner to select 8 fish. Be ready to defend your choices.

Tank Number One

<table>
<thead>
<tr>
<th>Fish Type</th>
<th>Length</th>
<th>Number of Fish</th>
</tr>
</thead>
<tbody>
<tr>
<td>Suckermouth Catfish</td>
<td>$\frac{4}{4}$ ft.</td>
<td></td>
</tr>
<tr>
<td>Red Cap Oranda Goldfish</td>
<td>$\frac{3}{4}$ ft.</td>
<td></td>
</tr>
<tr>
<td>Silver Dollar</td>
<td>$\frac{1}{2}$ ft.</td>
<td></td>
</tr>
<tr>
<td>Angelfish</td>
<td>$\frac{1}{2}$ ft.</td>
<td></td>
</tr>
<tr>
<td>Tiger Barb</td>
<td>$\frac{1}{4}$ ft.</td>
<td></td>
</tr>
<tr>
<td>Congo Tetra</td>
<td>$\frac{1}{4}$ ft.</td>
<td></td>
</tr>
</tbody>
</table>
Length of Fish, Line Plot Data Analysis

1. Create a line plot to represent the length of the fish chosen by the class. Be sure to label the x-axis and title the line plot.

   Title ________________________________

   _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ _______ ________________________________

   (label unit)

2. Find the median fish length.

3. Find the range of the fish lengths.

4. What is the mode of the fish lengths?

5. What is the total length of all of the fish combined?

6. What is the average (mean) length of these fish?
Second Tank Needs Fish

The school secretary has chosen the following combination of 8 fish for the second fish tank in the office.

<table>
<thead>
<tr>
<th>Fish Type</th>
<th>Length</th>
<th>Number of Fish</th>
</tr>
</thead>
<tbody>
<tr>
<td>Suckermouth Catfish</td>
<td>$\frac{3}{4}$ ft.</td>
<td>2</td>
</tr>
<tr>
<td>Red Cap Oranda Goldfish</td>
<td>$\frac{3}{4}$ ft.</td>
<td>1</td>
</tr>
<tr>
<td>Silver Dollar</td>
<td>$\frac{1}{2}$ ft.</td>
<td>2</td>
</tr>
<tr>
<td>Angelfish</td>
<td>$\frac{1}{2}$ ft.</td>
<td>2</td>
</tr>
<tr>
<td>Tiger Barb</td>
<td>$\frac{1}{4}$ ft.</td>
<td>0</td>
</tr>
<tr>
<td>Congo Tetra</td>
<td>$\frac{1}{4}$ ft.</td>
<td>1</td>
</tr>
</tbody>
</table>

1. With a partner, create a line plot showing the fish lengths. Be sure to title the line plot and label the appropriate units.

Title _________________________________________________________

0 __________  $\frac{1}{4}$ __________  $\frac{1}{2}$ __________  $\frac{3}{4}$ __________  $\frac{4}{4}$ __________

(label unit)

(Continued on next page.)
Second Tank Needs Fish (cont.)

2 Find the median fish length.

3 Find the range of the fish lengths.

4 Find the mode of the fish lengths.

5 What is the average (mean) length of the fish in feet?

CHALLENGE

6 What is the difference in the fish lengths? Compare the median, range, mode and mean.

7 What is the total length of all of the fish combined in feet?

8 Create a new set of data, different from the second fish tank, which has the same mean and median as the second fish tank. Record the data below on the line plot and give this data a title and appropriate unit label.

Title _________________________________________________________________________

______________________________ (label unit)
Set D2 ★ Independent Worksheet 4

Third Tank Needs Fish

Your principal went to the fish store and purchased different fish for the office fish tanks. The prices are shown on the line plot below. Solve the following problems.

Cost of Fish

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Purchased Fish (in dollars)</td>
<td>3</td>
<td>$3\frac{1}{4}$</td>
<td>$3\frac{1}{2}$</td>
<td>$3\frac{3}{4}$</td>
</tr>
</tbody>
</table>

1 How many fish did the principal purchase? __________

2 What was the difference in cost between the most expensive and least expensive fish?

3 Which fish price cost was the mode?

4 What was the total amount of money spent on the fish? __________

5 What was the average (mean) cost the principal spent on a fish?
Set D2 Activity 5

Fish Tank Plants with a Line Plot

Overview
Students interpret and analyze line plot data to determine the median, range, mode, and mean, in fractional units, with unlike denominators. They also discover what happens with a “0” value. Students also create a visual fraction model from paper strips to display a set of measurements.

You’ll need
★ Fish Tank Plants (pages D2.28 and D2.29, run a class set, plus 1 for display)
★ Plants for the First Fish Tank (page D2.30, run 1 copy for display)
★ Student journals or Journal Grid Page (page D2.31, run as needed, optional)
★ Class set of 1½” × 12” construction paper strips in any color

Advance Preparations Cut a class set of 1½” × 12” long strips of paper (any color)

Skills & Concepts
★ Solve word problems involving addition and subtraction of fractions using visual fraction models or equations to represent the problem.
★ Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers.
★ Solve word problems involving division of whole numbers using visual fraction models or equations to represent the problem.
★ Solve real world problems involving multiplication of fractions and mixed numbers, using visual fraction models or equations to represent the problem.
★ Convert among different-sized standard measurement units within a given measurement system.
★ Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8).
★ Use operations on fractions for this grade to solve problems involving information presented in line plots

Instructions for Fish Tank Plants
1. Display the Fish Tank Plants master and have students read the scenario. Ask the class to tell a partner what information is given in the problem. What information is missing?

2. Discuss with the class what each symbol represents within the line plot display. Students might notice the “X” symbol represents the seeds that have not grown yet. If they don’t make this connection yet, don’t be concerned. Question 3 will ask the class to clarify the value of 0.
3. Students might notice that the fractions have unlike denominators. Invite them to consider how this is different than the line plots they have previously seen. Have students write an equivalent fraction for the denominators below the fourths and halves, and then fill in the missing values on their record sheet.

4. Work together as a class to solve questions 1–6 with the Plant Leaf Width line plot. For question 7, provide some additional time for students to record an answer. Then invite volunteers to come up and share their work.

Jason I found the total leaf widths by adding all of the fractions

\[
\frac{1}{8} + \frac{1}{8} + \frac{3}{8} + \frac{3}{8} + \frac{5}{8} + \frac{5}{8} + \frac{3}{4} = \frac{21}{12}
\]

Bryer I respectfully disagree Jason. The denominators need to be the same before adding all of these fractions. Your problem should have said

\[
\frac{1}{8} + \frac{1}{8} + \frac{3}{8} + \frac{3}{8} + \frac{5}{8} + \frac{5}{8} + \frac{6}{8} = \frac{24}{8}
\]

Sammy I agree with you Bryer, but I knew I could reduce \( \frac{24}{8} = 3 \)

Tracy Bryer, what about adding the 0s to this equation? I thought that you need to show them.

Bryer I didn’t add the 0s because 5 zeros added still equals 0 and that would not change the equation.

Teacher What would an equation or expression look like if it matched the line plot? Do the zeros need to be added?

Sammy I think the 5 zeros should be part of the equation. Without them how would we show that there are 5 additional plants on this line plot?

5. Encourage students to also record an equation to show their thinking.

6. Question 9 asks the student to find the average weight of the plants and explain their thinking. One pound divided by twelve plants would be \( \frac{1}{12} \) of a pound.

7. Display the Plants for the First Fish Tank master while students access their journals. Read the top half of the directions as a class. Ask the class to turn to a partner and restate what they are being asked to do. Next, have students get into groups of 4–8 students to share the work for this task. You may want to consider grouping your students strategically.

8. Distribute the 12" long construction paper strips and ask students to construct a model for eighths. Students can achieve this by either folding the strips into eighths (half, half and half again) or by dividing 8 into 12 and measuring out the strip into 1 ½ inch sections. Students will use this visual model to demonstrate an understanding of each question.

9. Circulate while students work in teams, to observe and provide clarification. Let students know they can cut the strips and re-arrange them to find the mean (average).

10. When the majority of students are done, share the models and strategies students used. Invite students to ask questions about this work, agreeing or disagreeing respectfully about the written and oral explanations. A great way to do this is to compare two different explanations. What is the same and
what is different about the work? What information would we use to prove a correct solution for finding the mean?

Assign Independent Worksheet 5, Plants for the Second Fish Tank (page D2.45) for more practice with line plots.
Fish Tank Plants

All of the fish tanks are in need of some plants. The office staff has picked out 5 different plants to place in the tanks. One type of plant was in seed form and had no leaves yet. The line plot below represents the width of the plants leaves and how many of each type of plant were purchased.

Fish Tank Plants

<table>
<thead>
<tr>
<th></th>
<th>X</th>
<th>X</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
<td></td>
<td>1/8</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1/4</td>
<td></td>
<td></td>
<td></td>
<td>1/2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3/8</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4/4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Plant Leaf Width (in inches)

Use the information in the line plot to answer the questions below.

1. How many plants in all did the office purchase?

2. How many plants were purchased that measured $\frac{3}{8}$ of an inch in width?

3. How many plants were purchased in seed form?

4. What is the median plant leaf width?

5. Find the mode of the plant leaf widths.

(Continued on next page.)
Fish Tank Plants (cont.)

6 What is the difference between the widest leaf and the narrowest leaf?

7 What is the total width of the combined leaves?

8 What is the average (mean) width of the plants?

9 The secretary weighed the plants and their total weight was 1 pound.
   a What would the average weight of each plant be? (Not including the seeds.)

   b Explain your answer.
Plants for the First Fish Tank

When the tank needs a cleaning every two weeks, the school secretary would like 8 of the plants to be trimmed to the following lengths (fractions of a foot).

Directions
- Each team member is in charge of cutting 1 or 2 paper strips to match the lengths above. Use the strips to create a line plot, including a title, labeled axis and units.
- Then answer the questions in your journal. Use your pieces to prove your thinking.

Questions
1. How many plants needed to be trimmed to \( \frac{3}{4} \) of a foot?

2. What is the mode of this data set?

3. What is the median of the plant lengths?

4. Find the range of the data.

5. What is the total length of the plants combined?

6. What is the average (mean) length of the plants?
Journal Page Grid
Plants for the Second Fish Tank

The principal went to the pet store and purchased these plants at the prices shown below. These plant prices are displayed in fractions of a dollar. The store owner donated a few extra plants to the school. These donated plants cost the school nothing.

\[
\begin{array}{cccccccc}
\frac{1}{2} & 0 & \frac{1}{4} & \frac{3}{4} & 0 & 0 & 0 & \frac{1}{4} & 0 \\
\end{array}
\]

1. Use the data to create a line plot that has a proper title (including units), the axis labeled correctly, and the data plotted accurately.

Cost of Plants

---

---

---

---

---

---

(in ______________)

2. Use information on the line plot to help answer the following questions.

a. How many total plants did the principal walk out of the pet store with?

b. Not counting the free plants, which plant price was the mode?

(Continued on next page.)
Independent Worksheet 5  Plants for the Second Fish Tank (cont.)

What was the difference in cost between the most expensive and least expensive plant in the tank, not counting the free plants?

What was the total amount of money spent on the plants?

What was the average (mean) cost the principal spent on the plants?

If the bag of plants the principal purchased weighed 2 pounds, what would the average weight of each plant be? Explain your answer.
Set D2  ★  Activity 6

Water for the Fish Tank Assessment

Overview
Students work on a post-assessment to demonstrate proficiency with fraction computation, line plots and the mean of a data set. This assessment can be given along with the Unit Three Post Assessment.

Skills & Concepts
★ Solve word problems involving addition and subtraction of fractions using visual fraction models or equations to represent the problem.
★ Use operations on fractions for this grade to solve problems involving information presented in line plots
★ Interpret and analyze line plot data to determine the mean, in fractional units

You’ll Need
★ Water for the Fish Tank (page D2.34, run a class set)

Instructions for the Water for the Fish Tank Assessment
1. Give each student a copy of the Water for the Fish Tank Assessment and then read and review the tasks with the class.

2. Before students start to work, be sure they understand that they have to use numbers, labeled models, and/or words to show their work and/or explain their thinking; the answers alone will not be adequate.

3. Remind students that you are available to re-read any of the directions or problems for them while they work.
Water for the Fish Tank

A group of students were asked to fill a 20-gallon classroom fish tank with water. Each student was given a gallon container to fill. The line plot below shows the amount of water each student used to fill their container. Solve the following problems using the data.

1. How much more water would it take to fill the 20 gallon classroom fish tank?

2. If the water in the student gallon containers were redistributed equally, how much water would be in each container?

3. Two students arrive late to school and would like to contribute water to the 20 gallon tank. Each student contributed \( \frac{3}{4} \) of a gallon of water to the tank. What would be the new average (mean) contributed by the class? Explain.
Set A6: Number & Operations: Fraction Concepts
Activity 1: Simplify & Compare ........................................ A6.1
Ind. Worksheet 1: Using the Greatest Common Factor to Simplify Fractions .... A6.19
Activity 2: Same-Sized Pieces ............................................ A6.9
Ind. Worksheet 2: Finding the Least Common Denominator ..................... A6.21
Ind. Worksheet: LCM & GCF ................................................ A6.23

Set A11: Number & Operations: Multiplying & Dividing Decimals
Activity 1: Multiplying by Powers of Ten ................................ A11.1
Ind. Worksheet 1: Thinking about Tenths, Hundredths & Thousandths ........ A11.59
Activity 2: Dividing by Powers of Ten .................................. A11.7
Ind. Worksheet 2: Very Large & Very Small Numbers ....................... A11.61
Activity 3: Using Decimals to Calculate Sale Prices ........................ A11.15
Ind. Worksheet 3: Multiplying & Dividing by Powers of Ten ................ A11.63
Activity 4: Multiplying Decimals .......................................... A11.21
Ind. Worksheet 4: Using Landmark Fractions & Percents to Multiply by Decimals ... A11.65
Activity 5: Building a Deck, Using Partial Products & Arrays for Decimal
Multiplication ................................................................. A11.29
Activity 6: Multiplying Decimals, More/Less ................................ A11.35
Ind. Worksheet 5: Multiplying Two Decimals Numbers ....................... A11.67
Activity 7: Dividing Decimals with Money & Menus ........................ A11.45
Ind. Worksheet 6: Comparing & Multiplying Fractions & Decimals ........ A11.69
Activity 8: Using Models & Strategies to Divide with Decimals ................ A11.53
Ind. Worksheet 7: Olympic Swimmers .................................. A11.71
Ind. Worksheet 8: Olympic Track Star .................................. A11.73
Set A6 ★ Activity 1

Simplify & Compare

Overview
During this activity, students learn to simplify fractions by finding the greatest common factor of the numerator and the denominator. Then the teacher introduces a game to provide more practice with these new skills. Simplify & Compare can be used as a partner game once it has been introduced to the class, or played several times as a whole group.

Skills & Concepts
★ determine the greatest common factor of two whole numbers
★ simplify fractions using common factors

You’ll need
★ Simplify & Compare Game Board (page A6.7, run one copy on a transparency)
★ Simplify & Compare Record Sheets (page A6.8, run a class set)
★ students’ fraction kits (see Advance Preparation)
★ 1 1/2” x 12” construction paper strips, class set plus a few extra in each of the following colors: white, light brown, purple, green, orange, pink, blue, and yellow
★ class set of 6” x 9” manila or legal size envelopes
★ class set of scissors
★ class set of rulers
★ overhead double spinner
★ a more/less cube
★ overhead pens

Advance Preparation: Making Construction Paper Fraction Kits
Give each student a set of 5 construction paper strips, one each in the following colors: white, light brown, purple, green, and orange. Reserve a set of strips for yourself as well. Holding up the white strip, label it with a 1 as students do the same on their white strips.

Ask students to fold their light brown strip in half and cut it along the fold line as you do the same with your light brown strip. Ask students to identify the value of these two pieces relative to the white strip. Then have them label each light brown piece 1/2.

Note  If some of your students are already quite proficient with fractions, you might increase the challenge level of this activity by asking them to predict the length in inches of each fractional part as they cut and fold their strips.

Now ask students to fold the purple strip in half and then in half again. Before they unfold the strip, ask students to pair-share the number of segments they’ll see and the value of each, relative to the white strip. Then ask them to unfold the strip, check their predictions, cut along the fold lines, and label each part, as you do the same with your purple strip.
Next, ask students to fold their green strip in half, in half again, and in half a third time. Before they unfold it, have them pair-share their ideas about how many segments they’ll see and how the size of each will compare to the white strip. Some students might believe there will be 8 segments, while others are equally convinced that there will be 6. In either case, ask students to explain their thinking, although there’s no need to reach consensus right now. When students unfold their green strips, they’ll see 8 segments. If there’s been debate beforehand, you might continue the discussion as students cut and label each of the green pieces.

**Teacher**  So we got 8 parts instead of 6, even though we only folded the green strip 3 times. Why is that?

**Students**  Because you can see when you fold it that it’s half the size of a purple piece. I think what’s doubling is the number of pieces. Every time you fold the strip, you get double the number of pieces you got the last time, like 2 is double 1, 4 is double 2, and 8 is double 4. So it is a doubling pattern, just different from how some of us thought.

Once they have cut out and labeled the eighths, ask students to consider how the purple pieces (the fourths) compare to the whole and half strips. Students’ responses may provide some sense of their current understandings (and misconceptions) about fractions.

**Students**  The purple ones, the fourths, are half the size of the halves. Yeah, a fourth is like half of a half. Right! It’s like a half folded in half again. If you put 2 of the fourths together, they’re the same as a half.

**Teacher**  That’s very interesting. So how could we complete this equation? \( 1/4 + 1/4 = \)

**Students**  It’s \( 1/2 \). You can see the answer if you put 2 of the purples together.

**Teacher**  I’ve had students tell me the answer is \( 2/8 \). What do you think of that?

**Students**  Maybe they didn’t understand about fractions. Maybe they didn’t have these strips to look at. I know what they did. They added the numbers on top and the numbers on the bottom.

**Teacher**  Why doesn’t it work to do it that way?

**Students**  It’s hard to explain. I think fractions don’t work the same as regular numbers. I think it’s because they’re pieces, like parts of something else. I mean, if you added 2 of the white strips together, you’d get 2 because 1 + 1 is 2. But if you add 2 fourths together, it makes a larger piece—a half. And if you show two-eighths, two of the green pieces together, you can see it’s not the same as one-fourth plus one-fourth.

Now ask students to fold their orange strip in half 4 times. Again, ask them to make predictions about the number of segments they’ll see when they unfold the strip and how big each segment will be relative to the others they’ve cut and labeled. After a bit of discussion, have them cut the orange strip along the folds and label each piece.
Finally, ask students to work in pairs to arrange one of their sets as shown on the next page. Give them a couple minutes to pair-share mathematical observations about the pieces, and then invite volunteers to share their thinking with the class.

**Students**  The number of pieces in each row doubles. It goes 1, 2, 4, 8, then 16.
Whatever the number is on the bottom, that's how many there are of that piece, like there are 4 fourths, 8 eighths, and 16 sixteenths.
And they all match up. You can see that 2 fourths make a half, 4 eighths make a half, and 8 sixteenths make a half.
Remember when you said that you had some kids who thought that if you added $\frac{1}{4} + \frac{1}{4}$ you'd get $\frac{2}{8}$? But you can see that $\frac{2}{8}$ is the same as $\frac{1}{4}$.
There's stuff that doesn't match up too, like there's no bigger piece that's exactly the same size as $\frac{3}{16}$ or $\frac{3}{8}$.

**Making Thirds, Sixths, and Twelfths to Add to the Fraction Kits**
Next, give each student a set of 3 new construction paper strips, one each in the following colors: pink, blue, and yellow. Ask students to use their rulers to find and mark thirds on the pink strip before they fold and cut. Then have them label each piece with the fraction $\frac{1}{3}$.

Now ask students to fold the blue strip in thirds and then in half. Before they unfold the strip, ask them to pair-share the number of segments they will see and the value of each relative to the white strip. Then ask them to unfold the strip, check their predictions, cut it along the fold lines, and label each part.

Finally, ask the students to describe and then try any methods they can devise to fold the yellow strip into twelfths. Let them experiment for a few minutes. Some students may reason that they will be able to make twelfths by folding the strip into thirds, then in half, and then in half again. Others may use their rulers, reasoning that if the length of the whole is 12 inches, each twelfth must be 1". Still others may work entirely by trial and error and will need an extra yellow strip or two. When they are finished, give students each an envelope to store all their fraction pieces. (It's fine to fold the white strip so it will fit.)
Instructions for Simplify & Compare
1. Explain that students are going to use their fraction kits to learn more about fractions and play a new game today. Have them take all the fraction strips out of their envelopes and stack them in neat piles by size on their desks.

2. Write the fraction 6/8 at the overhead. Read it with the students and ask them to build the fraction with their pieces. Then challenge them to lay out an equivalent fraction with fewer pieces, all the same size as one another. Most children will set out three fourths in response. If some students set out one half and one fourth, remind them that all the pieces in the equivalent fraction have to be the same size.

3. Ask students to share any observations they can make about the two sets of pieces. Record the equation 6/8 = 3/4 on the overhead, and have students return the pieces they have just used to their stacks. Then write 8/16, and have students show this fraction with their pieces. When most have finished, ask them to build all the equivalent fractions they can find, using only same-sized pieces for each one. Give them a minute to work and talk with one another, and then invite volunteers to share their results.

Students
I got 8/16, 4/8, 2/4, and 1/2.
They’re all the same as 1/2.
When you use bigger pieces, you don’t need as many.

4. Write a series of numbers and arrows on the board to represent the sequence. Ask students to pair-share any observations they can make about the sequence of fractions, and then have volunteers share their ideas with the class. Can they find and describe any patterns? How do the numbers relate to one another? Which requires the fewest pieces to build?

Students
The numbers on the top, the numerators, go 8, 4, 2, and 1. It’s like they keep getting cut in half.
It’s the same with the numbers on the bottom. 16 ÷ 2 is 8. 8 ÷ 2 is 4. 4 ÷ 2 is 2.
A half was the fastest way to build the fraction.
I knew 8/16 was a half to begin with because 8 is half of 16.
Every number on the top is half of the number on the bottom.

5. Explain that 1/2 is the simplest way to show 8/16 because the numerator (1) and denominator (2) have no common factors other than 1.
6. Remind students that a factor is a whole number that divides exactly into another number. One way people find factors is to think of the pairs of numbers that can be multiplied to make a third number. Work with input from the students to list the factors of 8 and 16.

Factors of 8 are 1, 2, 4, and 8. You can divide 8 by each of these numbers.

\[
\begin{align*}
1 \times 8 &= 8 \\
2 \times 4 &= 8
\end{align*}
\]

Factors of 16 are 1, 2, 4, 8, and 16. You can divide 16 by each of these numbers.

\[
\begin{align*}
1 \times 16 &= 16 \\
2 \times 8 &= 16 \\
4 \times 4 &= 16
\end{align*}
\]

7. Work with input from the class to identify and circle the factors 8 and 16 have in common: 1, 2, 4, and 8. Then draw students’ attention back to \(\frac{1}{2}\). What are the factors of 1 and 2? What factors do the two numbers have in common? Only 1, so there’s no way to simplify the fraction any further.

8. Explain that you can find the simplest form of a fraction by building it with the fewest number of pieces. But you can also simplify a fraction by identifying the greatest common factor, or the biggest number by which you can divide both the numerator and the denominator. Write \(\frac{12}{16}\) on the board. Can this fraction be simplified? Ask students to pair-share ideas about the largest number by which both 12 and 16 can be divided. When they have identified 4 as the greatest common factor of 12 and 16, record the operation shown below at the overhead, and ask students to confirm it with their pieces. Is it true that \(\frac{12}{16}\) cannot be built with any fewer pieces than \(\frac{3}{4}\)?

\[
\begin{align*}
\frac{12}{16} &= \frac{4}{4} \\
\frac{12}{16} &= \frac{4}{4} \\
\frac{1}{4} &= \frac{1}{4} \\
\frac{1}{4} &= \frac{1}{4}
\end{align*}
\]

9. Repeat step 8 with \(\frac{10}{12}, \frac{3}{16},\) and \(\frac{12}{6}\). Students will note that \(\frac{3}{16}\) cannot be simplified because 3 and 16 have no factors in common other than 1. They will also discover that \(\frac{12}{6}\) simplifies to \(\frac{3}{2}\) and then converts to a mixed number, \(1\frac{1}{2}\).

10. Now explain that you’re going to play a new game with students that will give them more opportunities to simplify fractions by finding the greatest common factor. Ask them to carefully re-stack all their fraction strips by size while you place the Simplify & Compare game board on display at the overhead. Give students a few moments to examine it quietly, and then read the game rules with the class. Explain that they are going to play as Team 2, and you will play as Team 1. You will play a trial round so everyone can learn the rules, and then play the whole game with them.

11. Place the double spinner overlay on top of the spinners, spin both, and record the results under “Team 1”. Work with students to simplify your fraction by finding the greatest common factor for the numerator and denominator. Invite them to check the results with their fraction pieces as well.

12. Invite a volunteer up to the overhead to spin for the class. Record the students’ fraction under “Team 2” and work with their input to simplify it. Then ask students to compare their fraction with yours. If they are not sure which fraction is greater, have them build both with their fraction pieces. Use a <, >,
or = sign to show the results. Then have a second volunteer roll the more/less cube to determine the winner. Circle the winning fraction on the overhead.

**Teacher** I really lucked out on this first trial. I thought you were going to win because \( \frac{3}{4} \) is greater than \( \frac{1}{2} \), but Kendra rolled “less” instead of “more”.

13. Once the trial round is completed, erase the overhead. Give students each a copy of the Simplify & Compare record sheet and play 6 rounds with the class. You will need to erase the overhead between each round, but students will have a record of the complete game on their sheets. At the end of the game, have students take turns rolling the more/less cube for each pair of fractions. Have them circle the winning fraction for each round, fill in the scoreboard on their papers, and determine the winning team. If any of the pairs of fractions are equal, both teams score a point for the round.

**Extensions**
- Play Simplify & Compare several times with the class. The game provides an engaging context in which to practice simplifying and comparing fractions, and you don’t have to play all 6 rounds at once.
- Run extra copies of the record sheet and game board, and have the students play the game in pairs. Encourage them to use their fraction kits to confirm their answers if necessary.

**INDEPENDENT WORKSHEET**

Use Set A6 Independent Worksheets 1 and 3 to provide students more practice simplifying fractions by finding the greatest common factor for the numerator and denominator.
Simplify & Compare Game Board

Take turns:

1. Spin the top spinner to get your numerator. Spin the bottom spinner to get your denominator.

2. Record your fraction. Simplify it if you can. Change it to a mixed number if it is greater than 1.

3. After each of you have had a turn, use a <, =, or > sign to compare the two fractions.

4. Play 6 rounds. Then roll a More/Less cube to see which team wins each round. Circle the winning fraction and mark a point for the correct team on the scoreboard each time.
## Simplify & Compare Record Sheet

<table>
<thead>
<tr>
<th>Round 1</th>
<th>Round 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Team 1</td>
<td>Team 2</td>
</tr>
<tr>
<td>Simplify and Compare</td>
<td>Simplify and Compare</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Round 3</th>
<th>Round 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Team 1</td>
<td>Team 2</td>
</tr>
<tr>
<td>Simplify and Compare</td>
<td>Simplify and Compare</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Round 5</th>
<th>Round 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Team 1</td>
<td>Team 2</td>
</tr>
<tr>
<td>Simplify and Compare</td>
<td>Simplify and Compare</td>
</tr>
</tbody>
</table>

### Scoreboard

<table>
<thead>
<tr>
<th>Team 1</th>
<th>Team 2</th>
</tr>
</thead>
</table>

© The Math Learning Center

A6.8 • Bridges in Mathematics Grade 5 Supplement
Using the Greatest Common Factor to Simplify Fractions

1 Write all the factors of each number below. Try to think of the factors in pairs.

- **ex.** \( \frac{4}{12} \)
  - Factors of the Numerator: 1, 2, 4
  - Factors of the Denominator: 1, 2, 3, 4, 6, 12
  - Greatest Common Factor: 4
  - Divide to Get the Simplest Form: \( \frac{4}{12} \div 4 = \frac{1}{3} \)

2 You can simplify a fraction by dividing the numerator and the denominator by the same number. If you divide the numerator and denominator by the largest factor they have in common (the greatest common factor), you can show the fraction in its simplest form. Look carefully at the example below. Then fill in the rest of the table.

<table>
<thead>
<tr>
<th>Fraction</th>
<th>Factors of the Numerator (Top Number)</th>
<th>Factors of the Denominator (Bottom Number)</th>
<th>Greatest Common Factor</th>
<th>Divide to Get the Simplest Form</th>
<th>Picture and Equation</th>
</tr>
</thead>
</table>
| **ex.**  | \( \frac{4}{12} \)                    | 1, 2, 4                                    | 4                     | \( \frac{4}{12} \div 4 = \frac{1}{3} \) | ![Diagram](4_12)
|          |                                       | 1, 2, 3, 4                                 |                       |                                 |                     |
|          |                                       | 4, 6, 12                                   |                       |                                 |                     |
| **a**    | \( \frac{8}{12} \)                    |                                           |                       |                                 |                     |
|          |                                       |                                           |                       |                                 |                     |
| **b**    | \( \frac{4}{6} \)                     |                                           |                       |                                 |                     |
|          |                                       |                                           |                       |                                 |                     |

(Continued on back.)
Independent Worksheet 1  Using the Greatest Common Factor to Simplify Fractions (cont.)

3 Find the greatest common factor of each pair of numbers below.

<table>
<thead>
<tr>
<th>Example</th>
<th>a  6 and 21</th>
</tr>
</thead>
<tbody>
<tr>
<td>6 and 16</td>
<td></td>
</tr>
<tr>
<td>Factors of 6</td>
<td>1, 2, 3, 6</td>
</tr>
<tr>
<td>Factors of 16</td>
<td>1, 2, 4, 8, 16</td>
</tr>
<tr>
<td>Greatest Common Factor of 6 and 16</td>
<td>2</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Example</th>
<th>b  8 and 24</th>
</tr>
</thead>
<tbody>
<tr>
<td>8 and 24</td>
<td></td>
</tr>
<tr>
<td>Factors of 8</td>
<td></td>
</tr>
<tr>
<td>Factors of 24</td>
<td></td>
</tr>
<tr>
<td>Greatest Common Factor of 8 and 24</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Example</th>
<th>c  18 and 24</th>
</tr>
</thead>
<tbody>
<tr>
<td>18 and 24</td>
<td></td>
</tr>
<tr>
<td>Factors of 18</td>
<td></td>
</tr>
<tr>
<td>Factors of 24</td>
<td></td>
</tr>
<tr>
<td>Greatest Common Factor of 18 and 24</td>
<td></td>
</tr>
</tbody>
</table>

4 Use your answers from problem 3 to simplify these fractions.

<table>
<thead>
<tr>
<th>Example</th>
<th>a  6/21</th>
</tr>
</thead>
<tbody>
<tr>
<td>6/21</td>
<td></td>
</tr>
<tr>
<td>12/16</td>
<td>3/8</td>
</tr>
<tr>
<td>6/16</td>
<td>3/8</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Example</th>
<th>b  8/24</th>
</tr>
</thead>
<tbody>
<tr>
<td>8/24</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Example</th>
<th>c  18/24</th>
</tr>
</thead>
<tbody>
<tr>
<td>18/24</td>
<td></td>
</tr>
</tbody>
</table>

5 A fraction is in its simplest form when its numerator and denominator have no common factor other than 1. Look at the fractions below.

- Circle the fractions that can be simplified.
- Put a line under the fractions that are already in simplest form.

\[
\frac{3}{6}, \frac{5}{8}, \frac{4}{10}, \frac{12}{15}, \frac{2}{7}, \frac{8}{14}, \frac{3}{13}
\]

6 Choose three of the fractions in problem 5 that can be simplified. Simplify them below. Show your work.
Set A6 ★ Activity 2

Same-Sized Pieces

Overview
Students use sketches to compare fractions with unlike denominators. The teacher then introduces the idea of finding the least common multiple to rewrite fractions so they have common denominators. Students practice and apply this skill as a whole group and then independently.

Skills & Concepts
★ find the difference between two fractions
★ find the least common multiple of two or more whole numbers
★ given two fractions with unlike denominators, rewrite the fractions with a common denominator

You’ll need
★ Square Sandwiches & Bedroom Walls (page A6.14, run one copy on a transparency)
★ Same-Sized Pieces (page A6.15, run a class set)
★ Fraction Equivalents Worksheet (pages A6.16 and A6.17 run a class set)
★ piece of paper to mask parts of the overhead
★ overhead pens
★ rulers

Instructions for Same-Sized Pieces
1. Let students know that during this activity the class is going to develop some strategies for comparing fractions. Then display the top portion of the Square Sandwiches overhead. Keep the bottom part of the overhead covered for now.

2. Read the text to the class and ask students to pair-share ideas about the situation. Is it true that Corey got more than Ben? Exactly how much more did Corey get? How might dividing each of the sandwiches into same-sized pieces help students solve the problem?
Students  A third is more than a fourth, but it's hard to tell how much more. If you could cut both of the sandwiches into smaller pieces, you could maybe count up the pieces to see how many more of them are in a third.
I don't get it!
I think we're supposed to figure out a way to cut the sandwiches so they both have the same number of pieces. Right now, the first sandwich is cut into 4 pieces. The second sandwich is cut into 3 pieces. How could we make more cuts so they both have the same number?

3. Give students each a copy of the Same-Sized Pieces blackline. Note with students that there are 2 copies of the sandwich squares so they can try at least two different ideas. Some children might want to cut out and fold the sandwich squares, while others may want to draw lines on the squares.

4. After they have had a few minutes to work ask students to share their thinking and compare their answers with neighbors. Then invite several volunteers to share their thinking at the overhead.

Nick  I saw that if you divide each section of the first sandwich up and down you would get 8 pieces so Ben got \(\frac{2}{8}\) of a sandwich. I divided the other sandwich with a line across and saw that you would get 6 pieces and two of those would be the same as the third, so Corey got \(\frac{2}{6}\) of a sandwich. Sixths are bigger than eighths, so \(\frac{2}{6}\) is more than \(\frac{2}{8}\).

Jade  But that still doesn't tell us how much more Corey got than Ben. I thought we were supposed to make both sandwiches into the same sized pieces.

Teacher  How did you solve the problem, Jade?

Jade  Well, I kind of thought about how fourths and thirds go together, and I realized you could cut both of the sandwiches into 12 pieces, like this.

Teacher  Let's look at the situation again. Should we end up with more pieces all the same size for each sandwich like Nick did or should we cut both sandwiches so they both have the same number of pieces, like Jade did? Talk to the person next to you about this.

Steven  It's easier to compare if both sandwiches are cut the same. I did the same thing as Jade. You can see that Corey got \(\frac{4}{12}\) of a sandwich, and Ben only got \(\frac{3}{12}\). Corey got \(\frac{1}{12}\) more than Ben did.
Activity 2  Same-Sized Pieces (cont.)

5. Summarize the sandwich situation by writing the following equations on the board or overhead. How do fourths, thirds, and twelfths relate to one another? Ask students to pair-share ideas, and then call on volunteers to share with the class.

\[
\frac{1}{4} = \frac{3}{12} \quad \frac{1}{3} = \frac{4}{12} \quad \frac{3}{12} < \frac{4}{12}
\]

**Students** If you can figure out how to make both things, like the sandwiches, into pieces that are the same, you can tell who has more. You can cut fourths and thirds into twelfths. 3 and 4 both go into 12. Also, you can get to 12 if you count by 3s and if you count by 4s.

**Teacher** So, 3 and 4 are both factors of 12, and 12 is a multiple of 3 and a multiple of 4.

6. Show the next problem on the overhead. Read it with the class and clarify the situation as needed.

7. Give students a few minutes to solve the problem by experimenting with the rectangles at the bottom of their Same-Sized Pieces blackline. Encourage children who finish quickly to generate a second, and even third solution. Ask them to check their ideas and solutions with others nearby, and then invite several volunteers to the overhead to share their thinking with the class.

**LaTonya** This is so cool! I just split the halves into thirds and the thirds into halves, and got sixths for both walls. Raven painted one more sixth of her wall.
Greg  I did sixths at first, and then I split them up into twelfths. Jasmine painted \(\frac{6}{12}\) of her wall, and Jasmine painted \(\frac{3}{12}\) of her wall.

Sam  I did the same thing as Sam, but I cut the pieces the other way.

8. Chances are, your students will discover that the amount of wall space each girl painted can be compared by cutting the rectangles into sixths, twelfths, perhaps even eighteenths or twenty-fourths. Summarize their findings by writing equations similar to the ones below on the board or overhead. Ask students to share their ideas about how sixths and twelfths relate to halves and thirds.

\[
\begin{align*}
\frac{1}{2} &= \frac{3}{6} \\
\frac{2}{3} &= \frac{4}{6} \\
\frac{3}{6} &< \frac{4}{6} \\
\frac{1}{2} &= \frac{6}{12} \\
\frac{2}{3} &= \frac{8}{12} \\
\frac{6}{12} &< \frac{8}{12}
\end{align*}
\]

9. Explain that in order to compare, add, or subtract fractions that have different denominators, such as \(\frac{1}{4}\) and \(\frac{1}{3}\) or \(\frac{1}{2}\) and \(\frac{2}{3}\), people usually rewrite both fractions so they have the same denominator. Most students will readily agree that rewriting \(\frac{1}{2}\) as \(\frac{3}{6}\) and rewriting \(\frac{2}{3}\) as \(\frac{4}{6}\) makes it possible to compare the two with complete accuracy. Furthermore, people usually look for the lowest or least common denominator; in this case sixths rather than twelfths, eighteenths, or twenty-fourths.

While it is possible to find the least common denominator for two fractions by dividing them into smaller pieces as students have been doing today, one can also find the least common denominator by finding the least common multiple of the denominators. Write \(\frac{1}{4}\) and \(\frac{1}{3}\) on the board. Work with student input to identify the denominators and find the least common multiple of 4 and 3 by skip counting. Record the work as shown below.

\[
\begin{align*}
\frac{1}{4} &\quad \frac{1}{3} \\
4, 8, 12 &\quad 3, 6, 9, 12 \\
12 \text{ is the least common multiple of 4 and 3.}
\end{align*}
\]

10. Ask students to consider what the equivalent of \(\frac{1}{4}\) and \(\frac{1}{3}\) would be in twelfths. How many twelfths are there in each of these fractions? Have them re-examine the squares they divided at the beginning of the activity. Then show them how to get the same results by multiplying the numerator and denominator of \(\frac{1}{4}\) and \(\frac{1}{3}\) by 3 and 4 respectively.

\[
\begin{align*}
\frac{1}{4} &\quad \frac{1}{3} \\
1 \times 3 &\quad 1 \times 4 \\
4 \times 3 &\quad 3 \times 4
\end{align*}
\]

11. Now write \(\frac{1}{2}\) and \(\frac{2}{3}\) on the board. Work with student input to find the least common multiple of 2 and 3, and then multiply the numerator and denominator of \(\frac{1}{2}\) by 3 and the numerator and denominator of \(\frac{2}{3}\) by 2.

\[
\begin{align*}
\frac{1 \times 3}{2 \times 3} &= \frac{3}{6} \\
\frac{2 \times 2}{3 \times 2} &= \frac{4}{6}
\end{align*}
\]
12. Write $\frac{1}{4}$ and $\frac{2}{6}$ on the board. Which of the two fractions is greater? Exactly how much greater? Ask students to work in pairs to find the least common multiple of 4 and 6, and use the information to rewrite $\frac{1}{4}$ and $\frac{2}{6}$ so they have a common denominator. After they have had a minute or two to work, ask volunteers to share their solutions and strategies with the class.

13. Repeat step 12 with two or three other pairs of fractions. Possibilities include $\frac{2}{6}$ and $\frac{3}{8}$, $\frac{3}{4}$ and $\frac{7}{12}$, and $\frac{3}{5}$ and $\frac{4}{6}$. Then give students each a copy of the Fraction Equivalents Worksheets. Review both sheets with the class and clarify as needed. When students understand what to do, have them go to work. Encourage them to help another, and circulate to provide help as needed. You might also want to give students a choice of working on the sheet independently, or working with you in a more supported small group setting.

Use Set A6 Independent Worksheets 2 and 3 to provide students more practice finding the difference between two fractions by rewriting them so they have common denominators.
Square Sandwiches & Bedroom Walls

1 Carlos had 2 extra square sandwiches. They were exactly the same size. He gave $\frac{1}{4}$ of the first sandwich to his friend Ben and $\frac{1}{3}$ of the second sandwich to his friend Corey.

Ben said, “Hey, that’s not fair! Corey got more than I did!”

Exactly how much more did Corey get? Divide each sandwich into same-sized pieces to find out.

2 Jasmine and Raven were painting 2 walls in Jasmine’s bedroom. The 2 walls were exactly the same size. Jasmine painted $\frac{1}{2}$ of the first wall. Raven painted $\frac{2}{3}$ of the other wall.

Exactly how much more did Raven paint than Jasmine? Divide each wall into same-sized pieces to find out. Is there more than one answer?
Same-Sized Pieces
For each of the following pairs of fractions, draw in lines so they have the same number of pieces. Then write the equivalent fraction name beside both.

**Example**

\[
\frac{1}{2} \quad \frac{3}{6} \quad \frac{1}{3} \quad \frac{2}{6}
\]

**a**

\[
\frac{1}{6} \quad \frac{1}{2}
\]

**b**

\[
\frac{3}{4} \quad \frac{2}{5}
\]

**c**

\[
\frac{2}{6} \quad \frac{3}{8}
\]
2 Teri and Jon each got a granola bar from their dad. Teri ate $\frac{3}{5}$ of hers. Jon ate $\frac{2}{7}$ of his. Who ate more? Exactly how much more? Use the rectangles below to help solve the problem. Show all of your work.

\[
\begin{array}{cccc}
& & & \\
\hline
\hline
\hline
\hline
\end{array}
\qquad
\begin{array}{cccc}
& & & \\
\hline
\hline
\hline
\hline
\end{array}
\]

\[
\text{Teri ate exactly } \frac{1}{10} \text{ more than } \frac{2}{7}.
\]

3 Ryan rode his bike $\frac{5}{6}$ of a mile. James rode his bike $\frac{7}{8}$ of a mile. Who rode farther? Exactly how much farther? Use the rectangles below to help solve the problem. Show all of your work.

\[
\begin{array}{cccc}
& & & \\
\hline
\hline
\hline
\hline
\end{array}
\qquad
\begin{array}{cccc}
& & & \\
\hline
\hline
\hline
\hline
\end{array}
\]

\[
\text{Ryan rode exactly } \frac{1}{24} \text{ more of a mile than James.}
\]

4 Find the least common multiple (LCM) of each pair of numbers.

<table>
<thead>
<tr>
<th>ex.</th>
<th>a</th>
<th>b</th>
</tr>
</thead>
<tbody>
<tr>
<td>6 and 8</td>
<td>3 and 5</td>
<td>4 and 5</td>
</tr>
<tr>
<td>6, 12, 18, 24</td>
<td>3, 6, 9, 12</td>
<td>4, 8, 10, 12</td>
</tr>
<tr>
<td>8, 16, 24</td>
<td>3, 5, 10</td>
<td>4, 8, 12</td>
</tr>
<tr>
<td>$24$ is the LCM of 6 and 8</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

5 Circle the fraction you think is greater in each pair. Then find out for sure by rewriting the fractions so they have common denominators. (Hint: Use the information from problem 4 to help. Put a star by the fraction that turns out to be greater.)

<table>
<thead>
<tr>
<th>ex.</th>
<th>a</th>
<th>b</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{3}{8}$</td>
<td>$\frac{2}{6}$</td>
<td>$\frac{1}{4}$</td>
</tr>
<tr>
<td>$\frac{3 \times 3}{8 \times 3} = \frac{9}{24}$</td>
<td>$\frac{2}{3}$</td>
<td>$\frac{2}{5}$</td>
</tr>
<tr>
<td>$\frac{2 \times 4}{6 \times 4} = \frac{8}{24}$</td>
<td>$\frac{4}{5}$</td>
<td></td>
</tr>
</tbody>
</table>
Finding the Least Common Denominator

Which is greater, $\frac{2}{3}$ or $\frac{4}{5}$? Exactly how much difference is there between these two fractions? If you want to compare, add, or subtract two fractions, it is easier if you rewrite them so they both have the same denominator.

To do this:

- Find the least common multiple of the denominators of the fractions.

  multiples of 3 3, 6, 9, 12, 15
  multiples of 5 5, 10, 15
  The least common multiple of 3 and 5 is 15.

- Multiply the numerator and denominator of each fraction by the same number so the denominators are equal.

  \[
  \frac{2 \times 5}{3 \times 5} = \frac{10}{15} \quad \frac{4 \times 3}{5 \times 3} = \frac{12}{15} \quad \frac{4}{5} \text{ is greater than } \frac{2}{3} \text{ by exactly } \frac{2}{15}
  \]

1. Find the least common multiple (LCM) of each pair of numbers.

   **ex.** 4 and 10
   
   4, 8, 12, 16, 20
   10, 20
   20 is the LCM of 4 and 10

   **a** 5 and 6

   **b** 2 and 7

2. Circle the fraction you think is greater in each pair. Then find out for sure by rewriting the fractions so they have common denominators. Hint: Use the information from problem 1 to help. Put a star by the fraction that turns out to be greater.

   **ex.**

   \[
   \frac{3 \times 5}{4 \times 5} = \frac{15}{20} \quad \frac{7 \times 2}{10 \times 2} = \frac{14}{20}
   \]

   **a** \[
   \frac{4}{5} \quad \frac{5}{6}
   \]

   **b** \[
   \frac{1}{2} \quad \frac{4}{7}
   \]
3. Find the least common multiple (LCM) of each pair of numbers.

<table>
<thead>
<tr>
<th></th>
<th>a</th>
<th>b</th>
<th>c</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>5</td>
<td>10</td>
<td>6</td>
</tr>
<tr>
<td>b</td>
<td>6</td>
<td>9</td>
<td>7</td>
</tr>
<tr>
<td>c</td>
<td>5</td>
<td>7</td>
<td></td>
</tr>
</tbody>
</table>

4. Circle the fraction you think is greater in each pair. Then find out for sure by rewriting the fractions so they have common denominators. Hint: Use the information from problem 3 to help. Put a star by the fraction that turns out to be greater.

<table>
<thead>
<tr>
<th></th>
<th>a</th>
<th>b</th>
<th>c</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>2/5</td>
<td>3/10</td>
<td></td>
</tr>
<tr>
<td>b</td>
<td>4/6</td>
<td>7/9</td>
<td></td>
</tr>
<tr>
<td>c</td>
<td>4/5</td>
<td>5/7</td>
<td></td>
</tr>
</tbody>
</table>

5. Erica swam $\frac{6}{8}$ of a mile on Monday. She swam $\frac{10}{12}$ of a mile on Tuesday. Did she swim farther on Monday or Tuesday? Exactly how much farther? Use numbers, words, and/or labeled sketches to solve this problem. Show all your work.

Erica swam exactly _______ of a mile farther on ______________.
Set A6 ★ Independent Worksheet 3

**LCM & GCF**

1 Two grasshoppers are hopping up the stairs. Gary starts at the bottom and hops up 3 stairs at a time. First he lands on step 3, then step 6, and so on. Grace starts at the bottom and hops up 4 stairs at a time. First she lands on step 4, then step 8, and so on.

   a The staircase has 24 steps. On which steps will both grasshoppers land? Use labeled sketches, numbers, and/or words to solve the problem. Show your work.

   Both grasshoppers will land on steps ________________________________.

   b What is the first step on which both grasshoppers will land? ____________
   This is the least common multiple of 3 and 4.

2 Find the least common multiple (LCM) of each pair of numbers.

<table>
<thead>
<tr>
<th>ex. 6 and 8</th>
<th>a 4 and 9</th>
<th>b 5 and 8</th>
<th>c 6 and 14</th>
</tr>
</thead>
<tbody>
<tr>
<td>6, 12, 18, 24</td>
<td>6,</td>
<td>5,</td>
<td>6</td>
</tr>
<tr>
<td>8, 16, 24</td>
<td>12</td>
<td>8,</td>
<td>14</td>
</tr>
<tr>
<td>24 is the LCM</td>
<td>of 6 and 8</td>
<td>of 5 and 8</td>
<td>of 6 and 14</td>
</tr>
</tbody>
</table>

3 Circle the fraction you think is greater in each pair. Then find out for sure by rewriting the fractions so they have common denominators. Hint: Use the information from problem 2 to help. Put a star by the fraction that turns out to be greater.

<table>
<thead>
<tr>
<th>ex. ( \frac{5}{6} ) ¿ 6/8</th>
<th>a ( \frac{3}{4} ) ¿ 7/9</th>
<th>b ( \frac{2}{5} ) ¿ 3/8</th>
<th>c ( \frac{4}{6} ) ¿ 9/14</th>
</tr>
</thead>
<tbody>
<tr>
<td>5 x 4 = 20, 6 x 3 = 18</td>
<td>20, 18</td>
<td>20, 18</td>
<td>20, 18</td>
</tr>
<tr>
<td>6 x 4 = 24, 8 x 3 = 24</td>
<td>24, 24</td>
<td>24, 24</td>
<td>24, 24</td>
</tr>
</tbody>
</table>

(Continued on back.)
4 You can use the greatest common factor (GCF) to help simplify fractions.

Find the greatest common factor of each pair of numbers.

<table>
<thead>
<tr>
<th>ex.</th>
<th>12 and 24</th>
<th>a</th>
<th>8 and 20</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Factors of 12 are 1, 2, 3, 4, 6, 12</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Factors of 24 are 1, 2, 3, 4, 6, 8, 12, 24</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>12 is the GCF of 12 and 24</td>
<td></td>
<td></td>
</tr>
<tr>
<td>b</td>
<td>12 and 18</td>
<td></td>
<td></td>
</tr>
<tr>
<td>c</td>
<td>10 and 15</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

5 Use your answers from problem 4 to simplify these fractions.

<table>
<thead>
<tr>
<th>ex.</th>
<th>( \frac{12}{24} )</th>
<th>a</th>
<th>( \frac{8}{20} )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \frac{12}{12} ) = ( \frac{1}{2} )</td>
<td></td>
<td>( \frac{8}{20} ) = ( \frac{2}{2} )</td>
</tr>
<tr>
<td>b</td>
<td>( \frac{12}{12} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>( \frac{18}{24} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>c</td>
<td>( \frac{10}{15} )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

6 Ebony got \( \frac{3}{4} \) of a yard of red ribbon and \( \frac{10}{12} \) of a yard of purple ribbon. Which piece of ribbon was longer? Exactly what fraction of a yard longer was it? Use numbers, words, and/or labeled sketches to solve this problem. Make sure your answer is in simplest form.

The _________ piece of ribbon was exactly _______ of a yard longer than the _________ piece of ribbon.
Set A11 ★ Activity 1

Multiplying by Powers of Ten

Overview
Students complete a string of calculations with fractions and decimals and then discuss the relationships among those calculations to build greater computational fluency and a stronger number sense with decimals. Then they explore what happens, and why, when they multiply by powers of 10 (0.01, 0.1, 1, 10, etc.).

Skills & Concepts
★ multiply by powers of 10, including 0.01, 0.1, 1, 10, 100, and 1,000
★ describe the effect of place value when multiplying whole numbers and decimals by 0.01, 0.1, 1, 10, 100, and 1,000
★ apply fraction and decimal equivalencies to solve problems

You’ll need
★ Patterns in Multiplying by Powers of Ten (pages A11.4 and A11.5, run 1 copy for display, plus a class set)
★ Multiplying by Powers of Ten Practice (page A11.6, run 1 copy for display, plus a class set)
★ base ten pieces for each pair of students, plus a set for display
★ Great Wall of Base Ten saved from Unit Six

Advance Preparation
Try to find some copies of Bridges Student Book pages 160 and 161, Fraction & Decimal Equivalents, which students completed in Unit Six, Session 10. You might also fill in Display Master 6.10, Fraction & Decimal Equivalencies, which you used in Session 12. Both of these resources may jog students’ memory of the fraction equivalents of common decimals in steps 1 and 3 below.

Instructions for Multiplying by Powers of Ten

1. Explain to students that they’re going to be multiplying decimal numbers in the next few days and that they’ll begin with powers of 10, like 0.1, 10, and 100. Write the following problems one at a time where students can see them (answers included in parentheses for your reference). Ask students to work in pairs for a minute or two to solve one problem at a time, and then have students share their answers and strategies as a whole group.
   - \( \frac{1}{2} \times 10 \) (5)
   - \( 0.5 \times 10 \) (5)
   - \( \frac{1}{4} \times 10 \) (2.5)
   - \( 0.25 \times 10 \) (2.5)
   - \( 0.75 \times 10 \) (7.5)

When they have solved all five problems, ask students to discuss the relationships they noticed among the problems. Students are likely to note equivalencies between \( \frac{1}{2} \) and 0.5, and between \( \frac{1}{4} \) and 0.25. They may also have noticed that they could halve half of 10 to find one-fourth of 10, and that three-fourths (0.75) is three times one-fourth. They might also notice that when multiplying a decimal number by 10, you move the decimal point one place to the right (e.g., \( 0.25 \times 10 = 2.5 \)).
Describing the relationships among the problems should help students begin to develop efficient strategies for computing with decimal numbers. Students will solve similar sets of problems at the beginning of each activity in this set.

2. Place Patterns in Multiplying by Powers of Ten on display and give each student a copy. Review the sheet with the class. Discuss the sample equations in each table and have students connect the elements of each equation to the problem situation. Also be sure students remember how to write each decimal (0.01 and 0.1) as a fraction. Invite them to refer to Bridges Student Book pages 160 and 161, Fraction and Decimal Equivalents, or a filled in copy of Display Master 6.10, Fraction and Decimal Equivalencies, if you were able to retrieve these resources from Unit Six.

3. Give students time to complete the sheet in pairs. Then reconvene the class as a whole group and open the discussion by asking what they noticed about multiplying by 0.01, 0.1, and 10. Discuss each multiplier one at a time, and encourage students to explain why the patterns they see (e.g., “When you multiply by 0.01, the decimal point moves two places to the left”) make sense. Encourage students to refer to the Great Wall of Base Ten and to use the base ten pieces to explain the patterns they see. Remember that when modeling decimals, the mat represents 1, the strip 0.10, and the unit 0.01.

Josie I saw when you multiply a number by 0.01, like in the first problem, you can just move the decimal point two places to the left like this. It works every time.

\[45 \times 0.01 = 0.45\]
\[45.0 \text{ becomes } 0.45\]

Teacher Why does it work? Can you use the Great Wall of Base Ten or these base ten pieces to explain?

Josie Well, 45 times one-hundredth is 45 hundredths. 40 hundredths is the same as four-tenths. That’s the .4 part of the answer. And 5 hundredths is just 5 hundredths. So it’s like each part of the first number gets a hundred times smaller: 40 becomes four-tenths and 5 becomes five-hundredths. Or you could just think 45 hundredths, really. That’s a hundred times smaller than 45.
4. After students have discussed the patterns that emerged when multiplying by 0.01, 0.1, and 10, give each student a copy of Multiplying by Powers of Ten Practice. Explain that they’ll complete it independently, and then select a couple of problems from the sheet to do together before asking students to work on their own.

**Extensions**

- If students finish early, ask them to turn their papers over and write problems for each other in this form:

  \[
  45 \times \text{______} = 0.045 \quad 45 \times \text{_______} = 4,500 \quad 45 \times \text{________} = 4.5
  \]

  Then they can trade papers and fill in the missing powers of 10 in each equation.

- Clarify the term “power of ten” using the Great Wall of Base Ten, and introduce exponent notation. A power of ten is a number resulting from multiplying 10 by itself any number of times. We use exponents to show how many times a number, in this case 10, is multiplied by itself. A negative exponent indicates a number less than 1 (a fraction or a decimal).

\[
1,000 = 10^3 \quad 100 = 10^2 \quad 10 = 10^1 \quad 1 = 10^0 \quad 0.1 = 10^{-1} \quad 0.01 = 10^{-2}
\]
Patterns in Multiplying by Powers of Ten, page 1 of 2

1a The post office sells one-cent stamps. Fill out the table below to show how much it would cost to buy different quantities of one-cent stamps.

<table>
<thead>
<tr>
<th>Number of Stamps</th>
<th>Decimal Equation</th>
<th>Fraction Equation</th>
<th>Total Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 stamp</td>
<td>$1 \times 0.01 = 0.01$</td>
<td>$1 \times \frac{1}{100} = \frac{1}{100}$</td>
<td>$0.01$</td>
</tr>
<tr>
<td>2 stamps</td>
<td>$2 \times 0.01 = 0.02$</td>
<td>$2 \times \frac{1}{100} = \frac{2}{100}$</td>
<td>$0.02$</td>
</tr>
<tr>
<td>10 stamps</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>20 stamps</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>45 stamps</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>321 stamps</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>404 stamps</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

b What do you notice about multiplying by 0.01?

2a Amelia feeds her pet lizard crickets. The pet store sells crickets for ten cents each. Fill out the table below to show how much it would cost to buy different quantities of crickets.

<table>
<thead>
<tr>
<th>Number of Crickets</th>
<th>Decimal Equation</th>
<th>Fraction Equation</th>
<th>Total Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 cricket</td>
<td>$1 \times 0.10 = 0.10$</td>
<td>$1 \times \frac{1}{10} = \frac{1}{10}$</td>
<td>$0.10$</td>
</tr>
<tr>
<td>2 crickets</td>
<td>$2 \times 0.10 = 0.20$</td>
<td>$2 \times \frac{1}{10} = \frac{2}{10}$</td>
<td>$0.20$</td>
</tr>
<tr>
<td>10 crickets</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>20 crickets</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(Continued on next page.)
Patterns in Multiplying by Powers of Ten, page 2 of 2 (cont.)

2a (cont.)

<table>
<thead>
<tr>
<th>Number of Crickets</th>
<th>Decimal Equation</th>
<th>Fraction Equation</th>
<th>Total Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>45 crickets</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>321 crickets</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>404 crickets</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

b What do you notice about multiplying by 0.10?

3a Alfonso’s company sells T-shirts to soccer teams. Each T-shirt costs ten dollars. Fill out the table below to show how much it would cost to buy different quantities of T-shirts.

<table>
<thead>
<tr>
<th>Number of Shirts</th>
<th>Equation</th>
<th>Total Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 shirt</td>
<td>1 × 10 = 10</td>
<td>$10</td>
</tr>
<tr>
<td>2 shirts</td>
<td>2 × 10 = 20</td>
<td>$20</td>
</tr>
<tr>
<td>10 shirts</td>
<td></td>
<td></td>
</tr>
<tr>
<td>20 shirts</td>
<td></td>
<td></td>
</tr>
<tr>
<td>45 shirts</td>
<td></td>
<td></td>
</tr>
<tr>
<td>321 shirts</td>
<td></td>
<td></td>
</tr>
<tr>
<td>404 shirts</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

b What do you notice about multiplying by 10?
Multiplying by Powers of Ten Practice

Complete the following equations.

106 × 0.01 = ______ 47 × 0.01 = ______ 3 × 0.01 = ______
0.6 × 0.01 = ______ 0.32 × 0.01 = ______ 0.1 × 0.01 = ______
10 × 0.01 = ______

452 × 0.1 = ______ 302 × 0.1 = ______ 64 × 0.1 = ______
0.9 × 0.1 = ______ 0.57 × 0.1 = ______ 0.04 × 0.1 = ______
0.1 × 0.1 = ______

360 × 10 = ______ 23 × 10 = ______ 4 × 10 = ______
0.7 × 10 = ______ 0.54 × 10 = ______ 0.01 × 10 = ______

0.32 × 100 = ______ 4.3 × 100 = ______ 4 × 100 = ______
45 × 100 = ______ 309 × 100 = ______ 0.1 × 100 = ______

0.17 × 1,000 = ______ 0.34 × 1,000 = ______ 9.6 × 1,000 = ______
603 × 1,000 = ______ 0.01 × 1,000 = ______
Thinking about Tenths, Hundredths & Thousandths

1. Write two fractions that are equal to each decimal number.
   - $0.1 = \underline{\hspace{1cm}}$ and $\underline{\hspace{1cm}}$
   - $0.01 = \underline{\hspace{1cm}}$ and $\underline{\hspace{1cm}}$
   - $0.001 = \underline{\hspace{1cm}}$ and $\underline{\hspace{1cm}}$
   - $0.05 = \underline{\hspace{1cm}}$ and $\underline{\hspace{1cm}}$

2. Complete the chart below.

<table>
<thead>
<tr>
<th>Number</th>
<th>0.1 less</th>
<th>0.1 greater</th>
<th>0.01 less</th>
<th>0.01 greater</th>
<th>0.001 less</th>
<th>0.001 greater</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.2</td>
<td>1.1</td>
<td>1.3</td>
<td>1.19</td>
<td>1.21</td>
<td>1.199</td>
<td>1.201</td>
</tr>
<tr>
<td>8.73</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4.06</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6.9</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.896</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

3. Round each number to the place shown to complete the chart below.

<table>
<thead>
<tr>
<th>Number</th>
<th>Nearest tenth (0.1) Look at the 0.01 place.</th>
<th>Nearest hundredth (0.01) Look at the 0.001 place.</th>
<th>Nearest thousandth (0.001) Look at the 0.0001 place.</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1629</td>
<td>$0.2$</td>
<td>$0.16$</td>
<td>$0.163$</td>
</tr>
<tr>
<td>0.9608</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.0274</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6.0085</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Set A11 ★ Activity 2

Dividing by Powers of Ten

Overview
Students complete a string of calculations with fractions and decimals and then discuss the relationships among those calculations to build greater computational fluency and a stronger number sense with decimals. Then they explore what happens, and why, when they divide by powers of 10 (0.01, 0.1, 1, 10, etc.).

Skills & Concepts
★ divide by powers of 10, including 0.01, 0.1, 1, 10, 100, and 1,000
★ describe the effect of place value when dividing whole numbers and decimals by 0.01, 0.1, 1, 10, 100, and 1,000
★ apply fraction and decimal equivalencies to solve problems

You’ll need
★ Patterns in Dividing by Powers of Ten (pages A11.10–A11.12, run 1 copy for display, plus a class set)
★ Dividing by Powers of Ten Practice (page A11.13, run 1 copy for display, plus a class set)
★ base ten pieces for each pair of students, plus a set for display
★ Great Wall of Base Ten saved from Unit Six

Instructions for Dividing by Powers of Ten
1. Write the following problems one at a time where students can see them (answers included in parentheses for your reference). Ask students to work in pairs for a minute or two to solve one problem at a time, and then have students share their answers and strategies as a whole group.
   - 10 × 0.1 (1)
   - 10 × 0.6 (6)
   - 600 × 0.01 (6)
   - 600 × 0.04 (24)
   - 40 × 0.8 (32)

2. When they have solved all five problems, ask students to discuss the relationships they noticed among the problems. Students are likely to note that multiplying by 0.1 is like dividing by 10, just as multiplying by 0.01 is like dividing by 100. With this in mind, they can solve 600 × 0.04, for example, in the following way: 600 ÷ 100 = 6 and 6 × 4 = 24.

3. Now explain to students that today they’re going to be dividing by powers of 10, like 0.1, 10, and 100. Place Patterns in Dividing by Powers of Ten on display and give each student a copy. Review the sheet with the class. Discuss the sample equations in each table and have students connect the elements of each equation to the problem situation. Also be sure students remember how to write each decimal as a fraction.
Patterns in Dividing by Powers of Ten, page 1 of 3

1a Alfonso’s company sells T-shirts to soccer teams. Each T-shirt costs ten dollars. If you spent $1030, how many shirts could you buy?

b Fill out the table below to show how many T-shirts you could buy with different amounts of money.

<table>
<thead>
<tr>
<th>Total Cost</th>
<th>Equation</th>
<th>Number of Shirts</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10</td>
<td>10 ÷ 10 = 1</td>
<td>1</td>
</tr>
<tr>
<td>$20</td>
<td>20 ÷ 10 = 2</td>
<td>2</td>
</tr>
<tr>
<td>$100</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$200</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$450</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$1020</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

c What do you notice about dividing by 10?

2a Amelia feeds her pet lizard crickets. The pet store sells crickets for ten cents each. If Amelia spent $1.30 on crickets last week, how many crickets did she buy?

b Fill out the table below to show how much it would cost to buy different quantities of crickets.

<table>
<thead>
<tr>
<th>Total Cost</th>
<th>Decimal Equation</th>
<th>Fraction Equation</th>
<th>Number of Crickets</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0.10</td>
<td>0.10 ÷ 0.10 = 1</td>
<td>1/10 ÷ 1/10 = 1</td>
<td>1 cricket</td>
</tr>
<tr>
<td>$0.20</td>
<td>0.20 ÷ 0.10 = 2</td>
<td>2/10 ÷ 1/10 = 2</td>
<td>2 crickets</td>
</tr>
<tr>
<td>$1.00</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$2.00</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$3.30</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$5.20</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

c What do you notice about dividing by 0.10?

3a The post office sells one-cent stamps. If you spent $2.08, how many one-cent stamps could you buy?

b Fill out the table below to show how many stamps you could buy with different amounts of money.

<table>
<thead>
<tr>
<th>Total Cost</th>
<th>Decimal Equation</th>
<th>Fraction Equation</th>
<th>Number of Stamps</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0.01</td>
<td>0.01 ÷ 0.01 = 1</td>
<td>1/100 ÷ 1/100 = 1</td>
<td>1 stamp</td>
</tr>
<tr>
<td>$0.02</td>
<td>0.02 ÷ 0.01 = 2</td>
<td>2/100 ÷ 1/100 = 2</td>
<td>2 stamps</td>
</tr>
<tr>
<td>$0.10</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$0.40</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

c What do you notice about dividing by 0.01?

As you review the sheet, discuss how to write the numbers that are greater than 1 as a fraction. In this case, students will probably find it most useful to write them as improper fractions. For example, they would write 2.47 as $\frac{247}{100}$ in the first table. This will probably make dividing by $\frac{1}{100}$ more sensible to them.

Now ask students to complete the sheet in pairs. Encourage them to use the base ten pieces to think about the problems if that helps. Then reconvene the class as a whole group and open the discussion by asking what they noticed about dividing by 0.01, 0.1, and 10. Discuss each divisor one at a time, and encourage students to explain why the patterns they see make sense. (e.g., “When you divide by 0.01, the decimal point moves two places to the right. That’s what happens when you multiply by 100 too!”) Invite students to refer to the Great Wall of Base Ten and to use the base ten pieces to explain the patterns they see. Remember that when modeling decimals, the mat represents 1, the strip 0.10, and the unit 0.01.

**Sydney** When you divide by a decimal number, it’s like multiplying by the reverse whole number, so you move the decimal point that many places to the right.

**Teacher** Please use the base ten pieces to show us what you mean and why this is true.

**Sydney** Well, think about these strips. They show 40. So if you divide by 0.1, it’s like asking, how many tenths in 40? There are 10 tenths in each little unit and 40 units altogether, so you go 10 $\times$ 40 = 400. So 40 $\div$ 0.1 = 400. 400 is like 40 with the decimal one place to the right.
Students’ verbal explanations will vary considerably in their clarity, so encourage them to show their thinking with base ten pieces and equations. This will allow you to get a clearer sense of what they understand and will make their explanations more comprehensible to other students.

5. After students have discussed the patterns that emerged when dividing by 0.01, 0.1, and 10, give each student a copy of Dividing by Powers of Ten Practice. Explain that they’ll complete it independently, and then select a couple of problems from the sheet to do together before asking students to work on their own.

**Extensions**

- If students finish early, ask them to turn their papers over and write problems for each other in this form:
  
  \[
  45 \div \underline{} = 0.045 \quad 45 \div \underline{} = 450 \quad 45 \div \underline{} = 4.5
  \]

  Then they can trade papers and fill in the missing powers of 10 in each equation.

- You might also consider asking them to write their problems in this form:
  
  \[
  45 \div 10 = 45 \times \underline{} \quad 45 \div 0.10 = 45 \times \underline{} \quad 45 \div 0.01 = 45 \times \underline{} \]

- Help students understand powers of 10 in a graphic way. The Molecular Expressions web site (see URL below) features a photographic display called Secret Worlds: The Universe Within that illustrates powers of 10 starting with the Milky Way, 10 million light years (\(10^{20}\) meters) from Earth. A series of photos move closer and closer to Earth, decreasing in distance by a power of 10 each time, until you reach a tall oak tree seen at a distance of 1 meter (\(10^0\) meter). The photos don’t stop there, however. The powers of 10 go negative as the series moves in the microscopic world of an oak leaf, and finally into a subatomic universe of electrons and protons.


**INDEPENDENT WORKSHEET**

Use Set A11 Independent Worksheets 1–3 on pages A11.59–A11.64 to provide students with more practice multiplying and dividing by powers of ten, as well as rounding and doing calculations with decimals.
Patterns in Dividing by Powers of Ten, page 1 of 3

1 Alfonso’s company sells T-shirts to soccer teams. Each T-shirt costs ten dollars.
   a If you spent $1030, how many shirts could you buy?

b Fill out the table below to show how many T-shirts you could buy with different amounts of money.

<table>
<thead>
<tr>
<th>Total Cost</th>
<th>Equation</th>
<th>Number of Shirts</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10</td>
<td>10 ÷ 10 = 1</td>
<td>1</td>
</tr>
<tr>
<td>$20</td>
<td>20 ÷ 10 = 2</td>
<td>2</td>
</tr>
<tr>
<td>$100</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$200</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$450</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$3210</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$1020</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

c What do you notice about dividing by 10?

2 Amelia feeds her pet lizard crickets. The pet store sells crickets for ten cents each.
   a If Amelia spent $1.30 on crickets last week, how many crickets did she buy?

(Continued on next page.)
b Fill out the table below to show how much it would cost to buy different quantities of crickets.

<table>
<thead>
<tr>
<th>Total Cost</th>
<th>Decimal Equation</th>
<th>Fraction Equation</th>
<th>Number of Crickets</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0.10</td>
<td>$0.10 \div 0.10 = 1$</td>
<td>$\frac{1}{10} \div \frac{1}{10} = 1$</td>
<td>1 cricket</td>
</tr>
<tr>
<td>$0.20</td>
<td>$0.20 \div 0.10 = 2$</td>
<td>$\frac{2}{10} \div \frac{1}{10} = 2$</td>
<td>2 crickets</td>
</tr>
<tr>
<td>$1.00</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$2.00</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$3.30</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$5.20</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

c What do you notice about dividing by 0.10?

3 The post office sells one-cent stamps.

a If you spent $2.08, how many one-cent stamps could you buy?

b Fill out the table below to show how many stamps you could buy with different amounts of money.

<table>
<thead>
<tr>
<th>Total Cost</th>
<th>Decimal Equation</th>
<th>Fraction Equation</th>
<th>Number of Stamps</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0.01</td>
<td>$0.01 \div 0.01 = 1$</td>
<td>$\frac{1}{100} \div \frac{1}{100} = 1$</td>
<td>1 stamp</td>
</tr>
<tr>
<td>$0.02</td>
<td>$0.02 \div 0.01 = 2$</td>
<td>$\frac{2}{100} \div \frac{1}{100} = 2$</td>
<td>2 stamps</td>
</tr>
<tr>
<td>$0.10</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$0.40</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(Continued on next page.)
Patterns in Dividing by Powers of Ten, page 3 of 3  (cont.)

3b  Cont.

<table>
<thead>
<tr>
<th>Total Cost</th>
<th>Decimal Equation</th>
<th>Fraction Equation</th>
<th>Number of Stamps</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0.86</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$2.47</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$3.05</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

C What do you notice about dividing by 0.01?
Dividing by Powers of Ten Practice

Complete the following equations.

\[
\begin{align*}
3000 \div 1000 &= \underline{3} \\
372 \div 1000 &= \underline{0.372} \\
0.03 \div 1000 &= \underline{0.00003} \\
2504 \div 1000 &= \underline{2.504} \\
0.6 \div 1000 &= \underline{0.0006} \\
0.08 \div 1000 &= \underline{0.00008} \\
900 \div 100 &= \underline{9} \\
406 \div 100 &= \underline{4.06} \\
3 \div 100 &= \underline{0.03} \\
3.2 \div 100 &= \underline{0.032} \\
0.08 \div 100 &= \underline{0.00008} \\
405 \div 10 &= \underline{40.5} \\
0.63 \div 10 &= \underline{0.063} \\
87 \div 0.1 &= \underline{870} \\
6 \div 0.1 &= \underline{60} \\
0.5 \div 0.1 &= \underline{5} \\
0.48 \div 0.1 &= \underline{4.8} \\
3 \div 0.01 &= \underline{300} \\
6.9 \div 0.01 &= \underline{690} \\
0.8 \div 0.01 &= \underline{80} \\
409 \div 0.01 &= \underline{40900}
\end{align*}
\]
Set A11 ★ Independent Worksheet 2

Very Large & Very Small Numbers in Context

1. A micrometer is one-millionth of a meter (0.000001 m): ten thousand times shorter than a centimeter (0.01 m). How many micrometers long is one edge of a centimeter cube?

2. The football team for the University of Tennessee, the Tennessee Volunteers, plays its home games in the Neyland Stadium in Knoxville, Tennessee. The stadium holds about 100,000 people. (Do an image search on the internet to see what this many people looks like.)

   a. How many stadiums would it take to hold one million people (a bit less than the number of people living in Dallas, Texas)?

   b. According to estimates, there are over 300 million people living in the United States. How many Neyland Stadiums would it take to hold 300 million people?

3. The table below shows the estimated population of different countries as of 2012. Round each number to complete the table.

<table>
<thead>
<tr>
<th>Country</th>
<th>Population</th>
<th>Nearest 1,000,000</th>
<th>Nearest 100,000</th>
<th>Nearest 10,000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Philippines</td>
<td>103,775,000</td>
<td>104,000,000</td>
<td>103,800,000</td>
<td>103,780,000</td>
</tr>
<tr>
<td>Iran</td>
<td>78,868,710</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>France</td>
<td>65,630,690</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>South Korea</td>
<td>48,860,500</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Argentina</td>
<td>42,192,490</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sudan</td>
<td>34,206,710</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Set A11 ★ Activity 3

Using Decimals to Calculate Sale Prices

Overview
As a whole group, students review how to find a sale price, as well as fraction, decimal, and percent equivalences. Then students work in pairs to complete a set of related problems. At the end of the activity, students share their strategies for solving some of the more difficult problems.

Skills & Concepts
★ multiply whole numbers and decimal numbers by decimal numbers to the hundredths place
★ apply fraction, decimal, and percent equivalencies to solve problems

You'll need
★ The Game Sale (pages A11.18 and A11.19, 1 copy for display, plus a class set)
★ Fraction, Decimal & Percent Number Line from Unit Six, Session 16 (See Advance Preparation.)

Advance Preparation
Find the Fraction, Decimal & Percent Number Line, which you created with the class in Unit Six, Session 16. If you no longer have it, make an enlarged photocopy of the picture on page 881, Bridges Teacher’s Guide, Vol. 3. You might also consider playing the Number Line Game from Unit Six, Session 16 if you think students will need a refresher on equivalent fractions, decimals, and percents.

Instructions for using Decimals to Calculate Sale Prices
1. Write the following problems one at a time where students can see them (answers included in parentheses for your reference). Ask students to work in pairs for a minute or two to solve one problem at a time, and then have students share their answers and strategies as a whole group.
   • \( \frac{1}{2} \times 28 \) (14)
   • 0.50 \( \times 28 \) (14)
   • \( \frac{1}{4} \times 28 \) (7)
   • 0.25 \( \times 28 \) (7)
   • 0.50 \( \times 0.08 \) (0.04)
   • 0.25 \( \times 0.08 \) (0.02)

2. When they have solved all six problems, ask students to discuss the relationships they noticed among the problems. Students are likely to note that \( \frac{1}{2} \) is equal to 0.50 and that \( \frac{1}{4} \) is equal to 0.25. They may also have solved 0.50 \( \times 0.08 \) by reasoning that half of eight-hundredths is four-hundredths (0.04) and then halved again to solve 0.25 \( \times 0.08 \). Such strategies show a good understanding of the relationship between fractions, decimals, and division.

3. Explain that today’s activity involves finding the sale prices of different items. Invite students to share some examples of things they have purchased on sale. How much did the item cost originally? How was the sale expressed: in terms of a new price or a certain amount off?

4. After students have shared some examples, ask them to imagine that a bike that originally cost $120 is on sale for 10% off. How could fractions and decimals help them think about the new price for the
bike? (Write the scenario on the board.) Ask students to think about it quietly and then talk to a partner about their ideas. After a few moments, invite partners to share their thoughts with the whole group. Be sure students are clear that they need to calculate the discount (the percent taken off) and then subtract it from the original price to find the sale price, unless, of course, they calculate $120 \times 0.90$ to find the sale price.

After they have shared some ideas, which will likely involve thinking about fractions and division, refer students to the Fraction, Decimal and Percent Number Line from Unit Six, Session 16. Explain that they can use this number line to refresh their memories of fractions, decimals, and percents that are equivalent during today's activity.

5. Place The Game Sale on display and give each student a copy. Review the sheet with the class. In particular, you'll need to discuss the idea of recording a decimal equation for each row. Students are likely to use what they know about fractions and division to solve each problem, but writing an equation with the discount expressed as a decimal will prompt them to connect their work to multiplication with decimal numbers.

6. Circulate around the room while students work on the sheets in pairs. Take time to provide support, and reconvene the class as a group to discuss some of the problems if more than a few children are confused. Watch how students are working, and think about which problems you'd like to discuss as a whole group.

7. When you have about 15 minutes left in the session, reconvene the class as a whole group to discuss students' strategies for solving a few select problems from the sheets. If you saw students using a valuable or noteworthy strategy, invite them to share their work with the class.
Extension

Invite students to work on some more challenging sale problems. For example:

- A cell phone was 10% off. The sale price was $90. What was the original price?
- A digital camera was 10% off. The sale price was $225. What was the original price?
- A jacket was 25% off. The sale price was $36. What was the original price?

Students will come up with a variety of ways to solve these problems. Here is an example of how a fifth grader might solve the last problem.

This big square is the original price of the jacket. 25 percent is one-fourth of the total. The rest of it is $36. That's the sale price. It's made up of three-fourths of the original price. So I divided $36 by 3 to see how much each part was worth. $12 is one-fourth of the total original price, so that makes the original price $48.
The Game Sale, page 1 of 2

1 Rosa owns a game store, and she wants to put some of the older games in the store on sale to sell them quickly.

a If Rosa marks a board game that costs $38.50 at 50% off, what will be the sale price of the board game?

b If Rosa marks the same board game at 10% off, what will be the sale price of the board game?

c If Rosa marks the same board game at 20% off, what will be the sale price of the board game?

d If Rosa marks the same board game at 30% off, what will be the sale price of the board game?

(Continued on next page.)
2 Fill out the table below to show what the sale price would be for some different items in Rosa's store if she marked them at different sale rates.

**a** A puzzle that is originally priced at $16.50

<table>
<thead>
<tr>
<th>Sale</th>
<th>Your work</th>
<th>Equation</th>
<th>New Price</th>
</tr>
</thead>
</table>
| 50% off | Half of 16.50 is 8.25 | 16.50 \times 0.50 = 8.25  \\
|        |                    | 16.50 - 8.25 = 8.25    | \$8.25    |
| 10% off |                  | 16.50 \times _____ = _____  \\
|        |                    | 16.50 - _____ = _____    |           |
| 20% off |                  | 16.50 \times _____ = _____  \\
|        |                    | 16.50 - _____ = _____    |           |
| 30% off |                  | 16.50 \times _____ = _____  \\
|        |                    | 16.50 - _____ = _____    |           |

**b** A video game that is originally priced at $64

<table>
<thead>
<tr>
<th>Sale</th>
<th>Your work</th>
<th>Equation</th>
<th>New Price</th>
</tr>
</thead>
</table>
| 50% off |                  | 64 \times _____ = _____  \\
|        |                    | 64 - _____ = _____      |           |
| 25% off |                  | 64 \times _____ = _____  \\
|        |                    | 64 - _____ = _____      |           |
| 10% off |                  | 64 \times _____ = _____  \\
|        |                    | 64 - _____ = _____      |           |
| 40% off |                  | 64 \times _____ = _____  \\
|        |                    | 64 - _____ = _____      |           |
Set A11 ★ Independent Worksheet 3

Multiplying & Dividing by Powers of Ten

1 Solve the multiplication problems below.

$$34 \times 0.01 = \underline{\hspace{2cm}}$$  $$34 \times 0.10 = \underline{\hspace{2cm}}$$  $$34 \times 1 = \underline{\hspace{2cm}}$$

$$34 \times 10 = \underline{\hspace{2cm}}$$  $$34 \times 100 = \underline{\hspace{2cm}}$$  $$34 \times 1,000 = \underline{\hspace{2cm}}$$

2 Solve the division problems below.

$$34 \div 0.01 = \underline{\hspace{2cm}}$$  $$34 \div 0.10 = \underline{\hspace{2cm}}$$  $$34 \div 1 = \underline{\hspace{2cm}}$$

$$34 \div 10 = \underline{\hspace{2cm}}$$  $$34 \div 100 = \underline{\hspace{2cm}}$$  $$34 \div 1,000 = \underline{\hspace{2cm}}$$

3 What patterns do you notice in the equations you completed above?

4 Solve the multiplication and division problems below.

$$62 \div 100 = \underline{\hspace{2cm}}$$  $$3.4 \times 1000 = \underline{\hspace{2cm}}$$  $$7.89 \div 0.10 = \underline{\hspace{2cm}}$$

$$0.43 \times 100 = \underline{\hspace{2cm}}$$  $$0.08 \times 0.01 = \underline{\hspace{2cm}}$$  $$123.05 \div 100 = \underline{\hspace{2cm}}$$

5 Ramon bought erasers shaped like animals to give away at Family Night at his school. Each eraser costs $0.10. If he spent $25.60, how many erasers did he buy?

a Write a division equation to represent this situation.

b Solve the problem using a strategy that makes sense to you. Show all your work.
Set A11 ★ Activity 4

Multiplying Decimals

Overview
Students complete a string of related decimal calculations and then discuss the relationships among those calculations. Then they find the area of a computer chip to think about multiplying two decimal numbers. Finally, students solve two more story problems that require them to multiply decimal numbers, as well as a few straight calculations in which they multiply two decimal numbers using an algorithm, an array, or both.

Skills & Concepts
★ multiply decimal numbers to the hundredths place in a variety of ways, including using models
★ estimate solutions to arithmetic problems in order to assess reasonableness of results

You'll need
★ Decimal Grid (page A11.26, run 2 copies for display, a double-sided class set, plus extra)
★ Area of a Computer Chip (page A11.27, run 1 copy for display)
★ Using the Area Model to Multiply Decimal Numbers (page A11.28, run 1 copy for display, plus a class set)

Instructions for Multiplying Decimals
1. Write the following problems one at a time where students can see them (answers included in parentheses for your reference). Ask students to work in pairs for a minute or two to solve one problem at a time, and then have students share their answers and strategies as a whole group.
   • 0.10 × 43 (4.3)
   • 0.20 × 43 (8.6)
   • 0.10 × 4.3 (0.43)
   • 0.30 × 4.3 (1.29)

2. When they have solved all four problems, ask students to discuss the relationships they noticed among the problems. Students are likely to note that they can use one-tenth of a number (0.10) to determine any number of tenths (e.g., 0.20 and 0.30 in this case) of that same number.

3. Now explain that mental calculations like the ones they’ve been doing for the past few days aren’t as helpful when multiplying certain combinations of decimal numbers. Today they’ll be using the area model to help multiply some less friendly decimal numbers.

4. Display a copy of the Decimal Grid on the projector and give each student a double-sided copy of the grid. Ask students what the dimensions of the square must be if the total area is 1. After students have identified each dimension as 1 linear unit, ask them to identify what length each division on the grid indicates. (The heavier lines show tenths of a linear unit, and the finest grid lines show hundredths of a linear unit.) Label your grid to show these lengths, and have students do the same.

5. Then have them identify the fraction of the total area represented by the larger and smaller squares (hundredths and ten thousandths, respectively). Then ask students to identify what portion of the grid
represents one-tenth and one-thousandth of the total area (a strip of 10 large squares and a strip of 10 small squares, respectively). Label these areas on your grid, and ask students to do the same.

6. Post the Area of a Computer Chip problem on the projector, and ask students to turn their papers over and draw an array on their second Decimal Grid to represent the problem.

Before students continue, ask them to check their work with a partner and then make an estimate of the total area of their array with their partner. Tell them to refer to their labeled Decimal Grids or the one you have displayed at the overhead as needed.

7. Now ask a volunteer to help you label the dimensions of the array on another projected copy of the Decimal Grid. When the dimensions have been correctly labeled, outline the array and make lines to show the partial products within the array.
8. Before continuing, ask students to share their estimates of the total area of the array. When they do, encourage them to justify their thinking, and help them write each estimate in the form of an inequality. (You may need to invite them to refer to their labeled grids to remind them how big each piece of the grid is.)

\[
0.44 \times 0.44 < 0.25 \quad 0.44 \times 0.44 > 0.16
\]

9. Now ask students to divide their arrays into partial products as you have on the projected Decimal Grid. Then give them time to work in pairs to find the total area of the computer chip. Circulate while they work to listen in on their conversations. Reconvene the class to clarify any confusion that may arise. Otherwise, let them work, and reconvene the group when most have finished.

10. Invite volunteers to help you label the partial products on the array, and then ask them how they found the total area.
11. After they have shared their strategies, ask what they notice about the process or result of this calculation. Students may be surprised or interested to find that the total area is considerably smaller in relation to the area of the unit than the dimensions are in relation to the linear unit. Encourage them to use the Decimal Grid to explore why this is so. (The linear unit has been divided just once for each dimension, but the area unit has been divided twice, once at each linear dimension.)

You might also ask them to investigate why the product goes to four decimal places when the dimensions each go to just two decimal places. You might also want to wait until students have completed a few more problems before investigating this phenomenon. If they can discern and explain some patterns related to where the decimal point goes in the product, they will be able to use the standard algorithm to multiply decimal numbers. Prompting them to estimate a reasonable answer before they calculate will also help students be able to place the decimal point in the products based on what makes sense for the numbers they are multiplying.

12. Now give each student a copy of Using the Area Model to Multiply Decimal Numbers and ask them to complete the problems in pairs. These problems require students to sketch an array for each problem, rather than use a Decimal Grid. If students seem to be having trouble with their sketches, gather everyone together as a group to make the sketches together before having them continue solving the problems in pairs. (If necessary, allow students who need extra support to make their sketches on Decimal Grid paper and attach them to the worksheet.) A sketch of each problem is shown below for your convenience.

Without the entire Decimal Grid, students may have difficulty determining the area of each partial product, struggling to recall whether each unit of area in a given region is one hundredth or one thousandth of the total, for example. Encourage them to break the numbers apart to apply the associative property and use what they know about multiplying by powers of 10. For example, students might calculate the area of the larger partial product in the first example below in one of the following ways:

\[
0.3 \times 0.6 = 3 \times 0.1 \times 0.6 \\
3 \times 0.1 \times 0.6 = 3 \times 0.06 \\
3 \times 0.06 = 0.18 \\
0.3 \times 0.6 = 3 \times 0.1 \times 6 \times 0.1 \\
3 \times 0.1 \times 6 \times 0.1 = 3 \times 6 \times 0.1 \times 0.1 \\
3 \times 6 \times 0.1 \times 0.1 = 18 \times 0.01 \\
18 \times 0.01 = 0.18 \\
18 \times 0.01 = 0.18
\]
Activity 4 Multiplying Decimals (cont.)

1.2 km by 0.63 km = 0.756 km²

13. Extend students’ work into a second day if needed. After watching them work, consider opening the second day’s lesson with a mini-lesson focusing on whatever elements of these calculations were most challenging for them the first day.

INDEPENDENT WORKSHEET

Use Set A11 Independent Worksheets 4 and 5 on pages A11.65–A11.68 to provide students with more practice multiplying decimals.
Decimal Grid
Area of a Computer Chip

A certain computer chip measures 0.44 by 0.44 inches. What is the total area of the computer chip?
Using the Area Model to Multiply Decimal Numbers

1. A piece of paper measures 0.3 m by 0.65 m.
   a. Estimate the total area of the piece of paper.
   b. Make a labeled sketch of the piece of paper and use it to calculate an exact answer.

2. The city park measures 1.2 km by 0.63 km.
   a. Estimate the total area of the park.
   b. Make a labeled sketch of the park and use it to calculate an exact answer.

3. Use an algorithm or sketch arrays on another piece of paper to find the products below.
   \[
   \begin{array}{cccc}
   1.6 & 1.5 & 4.5 & 2.43 \\
   \times 0.7 & \times 0.2 & \times 2.3 & \times 1.8 \\
   \end{array}
   \]
Set A11 ★ Independent Worksheet 4

Using Landmark Fractions & Percents to Multiply by Decimals

1 At morning assembly, the principal said that the number of students at the school would be increasing by 10% next year.

a If there are 260 students at the school this year, how many more students are coming to the school next year?

b How many students will be at the school altogether next year?

c If the number of students increased by 30% over the next three years, how many more students would be coming to the school?

d If the number of students increased by 25% over the next three years, how many more students would be coming to the school?

2 Look at your work above. Use it to complete the equations below.

\[260 \times 0.10 = \underline{26} \]
\[260 \times 0.30 = \underline{78} \]
\[260 \times 0.25 = \underline{65} \]

3 Complete the following equations.

\[430 \times 0.10 = \underline{43} \]
\[430 \times 0.20 = \underline{86} \]
\[430 \times 0.50 = \underline{215} \]
\[84 \times 0.01 = \underline{0.84} \]
\[84 \times 0.02 = \underline{1.68} \]
\[84 \times 0.06 = \underline{5.04} \]
\[72 \times 0.50 = \underline{36} \]
\[72 \times 0.25 = \underline{18} \]
\[72 \times 0.75 = \underline{54} \]
\[0.12 \times 0.50 = \underline{0.06} \]
\[0.12 \times 0.25 = \underline{0.03} \]
\[0.12 \times 0.10 = \underline{0.01} \]
Set A11 ★ Activity 5

ACTIVITY

Building a Deck Using Partial Products & Arrays for Decimal Multiplication

Overview
Students continue to share strategies for estimating and multiplying two decimal numbers in the context of building a deck. They become more comfortable sketching an array and using an algorithm as a written method for their reasoning.

Skills & Concepts
★ Multiply decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between multiplication and division (CCSS 5.NBT.7)
★ Relate the strategy to a written method and explain the reasoning used. (CCSS 5.NBT.7)

Instructions for Decimal Multiplication
1. Show the outline of a quick sketch for $2 \times 4.7$ and ask students to consider how much deck material you need to purchase if you want to build a deck that is 2 meters by 4.7 meters wide. Ask students to use the sketch to estimate a reasonable answer.

   **Teacher** Looking at this model, can you estimate a reasonable product?

   **Amelia** Sure, 4.7 is closer to 5 than it is to 4, and 2 times 5 is 10. Since it’s not quite 5, the area is going to be a little less than 10.

2. Then, ask students to find the area and record their thinking on their decimal grid page. After a minute or two, have students pair share, and then invite a few students to share their thinking with the class.

   **Francisco** I made a sketch and split it into two parts. Then I could see 2 times 4, which is 8, and 2 times 0.7, which is 1.4. I added those two areas together, 8 + 1.4, and got 9.4 square meters for the deck.
**Teacher** Let me see if I can record your thinking using equations.

\[
2 \times 4 = 8 \\
2 \times 0.7 = 1.4 \\
8 + 1.4 = 9.4 \text{ square meters}
\]

Here is another way we can write this:

\[
(2 \times 4) + (2 \times 0.7) = 8 + 1.4 = 9.4 \\
2 \text{ meters} \times 4.7 \text{ meters} = 9.4 \text{ square meters}
\]

What if we wrote the problem this way to solve it? Take a moment to work with this problem in your journals.

\[
\begin{array}{c}
4.7 \\
\times 2
\end{array}
\]

**Corbin** I tried solving just like a multiplication problem with whole numbers, but I wasn’t really sure what to do with the decimal point at first. I knew 7 times 2 was 14, and I wrote the 4 in the ones column and carried the 1 to the tens. Then 4 times 2 plus 1 was 9, so I had 94. I knew the answer had to be a little less than 10, so the answer was 9.4.

\[
\begin{array}{c}
4.7 \\
\times 2
\end{array}
\]

\[\phantom{4.7} \]

\[\phantom{4.7} \]

\[= 9.4\]

3. Let students know that you have a bit more space and could make the deck just a bit bigger. What if the deck was 2.1 by 4.7 meters? As students consider the extra region, ask them What size will the new region be? Would they prefer to think about it as one region or make two regions? Why?

**Teacher** How much area would I be adding to my deck? How do you know?

**Chloe** I sketched the area across the bottom of the first deck, and I could see that there were four tenths across the bottom plus another tenth of 0.7.

**Teacher** How did you figure out what a tenth of 0.7 was?

**Lilly** I remembered that whenever you multiply a number by 0.1, the product is ten times smaller. You can just move the decimal point one place to the left, so I did, and I got 0.07. The new part of the deck is 0.4 + 0.07 = 0.47 square meters.
Teacher That’s interesting. When we multiplied 2 × 4.7 our answer only included tenths, 9.4, but now that we’re multiplying 2.1 × 4.7 our answer includes hundredths. Why do you suppose that is?

Carter Now we have to multiply tenths by tenths, and that gives us hundredths.

Teacher Let’s record the problem with partial products and look at that.

4.7
\[ \times 2.1 \]
8 (2 × 4)
1.4 (2 × 0.7)
0.4 (0.1 × 4)
0.07 (0.1 × 0.7)
9.87 square meters

4. Ask students to imagine that the neighbor down the street has a deck, too. Ask students to sketch a rectangle in their student journals and label the sides 1.5 meters and 3.6 meters. Ask them to pair share an estimate of the area with a neighbor and then find the area of the deck.

5. Share both the array and algorithm for multiplying decimals and ask students to make a connection to the partial products in the algorithm and the area in the array. What do they notice?
Teacher Let’s take a look at the multiplication algorithm for a minute. Where can we find the products 90 and 450 in our sketch?

Students It’s kind of like when we multiplied whole numbers, only we didn’t write the decimals on the algorithm until the end, and that makes it kind of confusing. Yeah, the 90 is really 0.90 from 1.5 × 6, and the 450 is really 4.5 from 3 × 1.5. You just think of the array in two parts instead of four. So (3 × 1) + (3 × 0.5) = 4.5

6. Finally, pose one more deck design, with the dimensions 2.5 meters by 6.7 meters. Will this deck be larger or smaller? By how much? What would be a good estimate of the product or area?

7. Share both the array and algorithm for multiplying decimals and ask students to make a connection to the partial products in the algorithm and the area in the array. What do they notice?

8. Close the session by asking students to respond to the following prompt in their student journals: How is solving a decimal multiplication problem the same or different than solving a whole number multiplication problem?
Decimal Grid
Set A11 ★ Activity 6

Multiplying Decimals, More/Less

Overview
In this session, students play three rounds of Multiplying Decimals, More/Less to develop fluency with multiplication of decimals. The player with the combined largest products at the end of the game wins.

Skills & Concepts
★ Estimate the product of multiplying whole and decimal numbers to determine a reasonable answer.
★ Multiply decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between multiplication and division (CCSS 5.NBT.7)
★ Relate the strategy to a written method and explain the reasoning used. (CCSS 5.NBT.7)

You’ll Need
★ More/Less die, half class-set
★ Domino Cards (page A11.39–A11.41 run a half class set on cardstock. See note.)
★ Student Math Journal or Journal Grid Page (page A11.42 optional, run 1 copy for display plus additional copies as needed)
★ Decimal Grid (page A11.43, run 1 copy for display, plus a double-sided class set)

Note If you saved the half-class set of Domino cards from Supplement Set A9, Activity 6, please reuse them instead of creating additional sets.

Instructions for Multiplying Decimals, More/Less

Game 1 Demonstration
1. Introduce the game Multiplying Decimals, by playing one game against the class.
   • Roll the more/less die to determine if you are playing for the greatest product or the lesser product.
   • Create a t-chart for you and your opponent in a student journal page. Label one side for Player One (teacher) and the second side as Player Two (students).

   Teacher  I’m going to choose a student to roll the die to determine if they team with more or less wins the game, the total of our products will decide.

   Armando  It says LESS!

   Teacher  In the first round of this game, I’ll draw two domino cards. I’ll read them as a decimal, and then multiply my two decimals to determine the product. But, I’ll need to consider how I read the domino. For example, should I use 5.3 or 3.5? And 6.8 or 8.6? Hmm… if I want a smaller product how should I read the decimals? Think privately… now share with your partner. What should I do?
Maya You can make a quick estimate of the product. $5 \times 8$ would be 40, and $3 \times 6$ would be 18.

Cooper Yeah, if you want less, you better make the decimal 3.5 and 6.8.

Jude 6.8 is almost 7, so I think the product is going to be more than 21.

Teacher Ok, so I figured $3 \times 6.8 = 20.40$ and then $.5 \times 6.8 = 3.4$. I added 20.40 + 3.4 and I have a total of 23.80. Let’s see what the students get!

2. Invite a student to draw two domino cards and show them to the rest of the class. Give students a moment to configure the best decimal combinations depending on the more/less die. Have students estimate the product, pair share and then invite a few students to share their thinking.

Parker 8 and 2, 3 and 9… if we want the least, we better multiply 2.8 $\times$ 3.9.

3. Then, have students compute the total at their desks and share their strategies. If necessary, sketch the problem with student input onto decimal grid paper, or record the steps numerically.

Tarin I have an idea… 3.9 is close to 4, so that’s like 2.8 $\times$ 4 and then you subtract one tenth or .28 to get the final product.

Mason Wow, we are really winning. You have twice as much as we do!

4. Record your product and your partners. Which product is more/less?

5. Continue the game until three rounds are played. Compute the total products from all three rounds. The player with the least wins this round, but if the die rolled for more, the player with the greatest decimal product would win.
Game 2: Partners
6. Pass out the Domino Blackline from Supplement Set A9 Activity 6, or pass out a new set for students to cut out and use for this game.

7. Have students roll the more/less die, and play 3 rounds with their partner. Remind students to estimate the product before they compute the total. Will the total be more or less than their estimate? Why?

8. When three turns have been played out, have players compute their totals and the player with the greatest or lesser total wins the game.

Game 3: More Challenge
9. Introduce a variation of the game. Tell students that this new game is played much the same way, but this time one domino will be used to create a whole number while the second domino is used to create the decimal. The third domino will become the multiplier. The four-digit number will be multiplied by a two-digit decimal number. For example, 23.36 × 0.59
10. Roll the die to determine if you are playing for more or less.

11. Estimate the product and then compute the actual answer. Keep track of your work and your partners.

12. Add up the total from three rounds to determine the winner!

**Note:** You may want to save the domino cards and use them for additional Work Place practice.

---

**INDEPENDENT WORKSHEET**

See Supplement Set A11 Independent Worksheet 6 on page A11.69 for more practice multiplying decimals.
Domino Cards page 1 of 3
Domino Cards page 2 of 3
Domino Cards page 3 of 3
Journal Page Grid
Decimal Grid
Set A11 ★ Independent Worksheet 5

Multiplying Two Decimal Numbers

1. The memory card for Steve's camera measures 0.82 inches by 1.25 inches
   a. What do you estimate the total area of the memory card is?
   b. Find the exact area of the memory card. Show all your work. Fill in the array below if it helps you.

   c. What is the place value of the smallest unit of area in the array above?

2. Fill in an estimate and the exact answer for the problems below.

   a. Estimate ________
      
      0.40
      \times 0.56
      
      Exact Answer ________

   b. Estimate ________
      
      2.06
      \times 1.42
      
      Exact Answer ________

   c. Estimate ________
      
      3.7
      \times 0.28
      
      Exact Answer ________
Set A11 ★ Activity 7

Dividing Decimals with Money & Menus

Overview
Students use money value pieces and multiplication menus to solve division word problems with decimals and whole numbers. Additional exploration with the partial quotients method helps students make connections to the traditional algorithm for dividing whole numbers. These strategies develop place value understanding and reasoning about quotients with decimals.

Skills & Concepts
★ Estimate the product of multiplying decimal numbers to determine a reasonable answer.
★ Multiply and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between multiplication and division (CCSS5.NBT.7)
★ Relate the strategy to a written method and explain the reasoning used. (CCSS 5.NBT.7)

Instructions for Menus for Decimals
1. Display Dividing Decimals with Money and Menus and ask students to silently read the first problem and consider a reasonable estimate. Invite students to pair share, and then invite a few students to share their thinking with the class.

 Students I know that 20 cents times 6 would be $1.20, and that's close. And 25 cents would be too much because 25¢ × 6 is $1.50, so it's more like 22 or 23 cents.
2. Ask students to consider a menu of quotients that might help them solve for this division problem. At the display fill in the menu on the Dividing Decimal with Money and Menus master with the combinations that students suggest. Be sure the menu includes the multiplication facts below.

- $6 \times .10 = 0.60$
- $6 \times .20 = 1.20$
- $6 \times .01 = 0.06$
- $6 \times .05 = 0.30$

Then, give students a moment to re-think their estimates while you pass out money value pieces. Have students work out the problem with the money value pieces, and then invite students to share a few efficient strategies. During the discussion, continue to emphasize decimal place value in the context of money.

**Jackson** Since we said that $6 \times 0.20 = \$1.20$, I started by setting out ten of the 10¢ pieces in groups of two. Then I needed 18¢ more, and 18 divided by 6 is 3¢, so I gave each of the six groups 3¢ each. Each piece cost $0.23.

**Teacher** Did anyone think of it a different way?

**Piper** I did. I tried to make an array like we did when we did division with the base ten pieces. First I set out six 10¢ strips, and that was 60¢. Then I set out six more, and that was $1.20. I didn’t have enough 10¢ strips to do it again, so I used the 18 pennies, which gave each of the 6 rows 3¢ more.
3. Present Problem 2 and fill in the menu on the display master with students. Be sure the menu includes these combinations:

- $3 \times 0.10 = $0.30
- $3 \times 0.20 = $0.60
- $3 \times 0.50 = $1.50
- $3 \times 1.00 = $3.00
- $3 \times 2.00 = $6.00

4. Ask students to look at the information on the menu and consider what a reasonable estimate would be for the quotient.

   **Students** If it was $3.00, they would each get $1, and if it was $6.00 they’d each get $2.00. Since it’s in between, I think they’ll each get around $1.50.

   It’s going to be less than $2.00 but not much less—maybe like $1.80.

Give students time to solve the problem. After a few minutes, invite several students to share their thinking with the class. When students agree on the correct answer, record the decimal division equation $5.82 \div 3 = $1.94 where everyone can see.

   **Ajay** I knew each brother would get $1.00, and that left $2.82. I gave each brother 50¢, and then there was still $1.32, so I gave them each a quarter. Then I had $0.57, so I gave them each another 10¢ and that left 27¢. $3 \times 9$ is 27, so altogether each brother got $1.94.
**Student** I thought of it a different way. I knew that $5.82 was really close to $6.00, so I thought about the difference between them, which is $0.18. I knew if there was $6.00, each brother would get $2.00. I divided the 18¢ by 3 and took away 6¢ from $2.00, so each brother got $1.94.

5. Continue with Problem 3. Create a menu together and have students estimate before they solve for the quotient. Invite students to help you identify menu combinations that would be helpful for this problem, and be sure to include 6 × $1.00 and 6 × .50. Record $9.12 ÷ 6 = _____ where everyone can see.

**Note** At this point, modeling with the money value pieces may not as efficient as sketching an array model and using the menus to determine the partial quotients.

**Students** I thought about this problem like $1 × 6, so a bit more than a dollar.
I know that 12¢ ÷ 6 = 2¢. Maybe $1.02.

6. Before sending students out to work independently, with a partner or in a small group with you, remind students to estimate a reasonable solution, make a menu, and then choose a division strategy that makes sense to them. It might be using the money value pieces, a quick sketch, partial quotients or the traditional algorithm. Have the students use one method to solve for the word problem and a second method to double-check their thinking.

**INDEPENDENT WORKSHEET**

Use Independent Worksheet 6 on pages A11.71 and A11.72 anytime after Activity 7 for more practice estimating and dividing whole numbers and decimals.
1 A package of gum costs $1.38. There are 6 sticks in the pack. How much does each stick of gum cost?

\[ \text{× Menu for } \] 
\[ \square \times \square = \square \] 
\[ \square \times \square = \square \] 
\[ \square \times \square = \square \] 
\[ \square \times \square = \square \] 
\[ \square \times \square = \square \] 

2 Alexander and his two brothers went to the zoo with their grandpa. At the end of the day, he gave the boys all of his change. He had $5.82 in his pocket. How much money did each brother get if they shared the money equally?

\[ \text{× Menu for } \] 
\[ \square \times \square = \square \] 
\[ \square \times \square = \square \] 
\[ \square \times \square = \square \] 
\[ \square \times \square = \square \] 
\[ \square \times \square = \square \] 

(Continued on next page.)
Kait spent $9.12 on 6 granola bars. How much did she pay for each one?

\[ \frac{9.12}{6} = \] 

The cash register recorded $3.44 for 4 packages of markers. Kale was going to pay for one of the packages. How much does he owe?

\[ \frac{3.44}{4} = \]
5 A group of 12 girls went to see a new movie on its opening night. Altogether their entrance fees were $88.20. How much did each girl pay?

6 Eight families decided to chip in to buy their team's practice soccer balls. The bill was $103.92, which was half the regular price. How much should each family pay, if everyone pays a fair share?
Set A11 ★ Independent Worksheet 6

Comparing & Multiplying Fractions & Decimals

1 Use one of the following symbols to make each expression below true.

\[ > \text{ (greater than) } < \text{ (less than) } = \text{ (equal to) } \]

| ex | \( \frac{2}{4} \text{ } \bigg| \bigg. \frac{3}{5} \) | a | \( \frac{11}{16} \text{ } \bigg| \bigg. \frac{3}{4} \) | b | \( \frac{3}{4} \text{ } \bigg| \bigg. \frac{3}{5} \) |
|----|------------------|---|------------------|---|------------------|
| c | \( \frac{6}{14} \text{ } \bigg| \bigg. \frac{1}{2} \) | d | \( \frac{1}{2} \text{ } \bigg| \bigg. \frac{8}{32} \) | e | \( \frac{5}{7} \text{ } \bigg| \bigg. \frac{4}{9} \) |
| f | \( \frac{2}{3} \text{ } \bigg| \bigg. \frac{6}{9} \) | g | 0.34 \( \bigg| \bigg. \frac{1}{4} \) | h | 0.58 \( \bigg| \bigg. \frac{4}{5} \) |

2 Convert the decimal to a fraction and multiply. Write the product in the simplest form.

<table>
<thead>
<tr>
<th>a</th>
<th>0.25 ( \times ) 13 = __________</th>
<th>b</th>
<th>57 ( \times ) 0.50 = __________</th>
<th>c</th>
<th>23 ( \times ) 0.25 = __________</th>
</tr>
</thead>
<tbody>
<tr>
<td>d</td>
<td>0.25 ( \times ) 27 = __________</td>
<td>e</td>
<td>56 ( \times ) 0.25 = __________</td>
<td>f</td>
<td>37 ( \times ) 0.75 = __________</td>
</tr>
<tr>
<td>g</td>
<td>3 ( \times ) 0.25 = __________</td>
<td>h</td>
<td>0.25 ( \times ) 7 = __________</td>
<td>i</td>
<td>8 ( \times ) 0.50 = __________</td>
</tr>
</tbody>
</table>
Set A11 ★ Activity 8

Using Models & Strategies to Divide with Decimals

**Overview**
Students use sketches, menus, the partial quotient and traditional algorithm to solve word problems with decimals and whole numbers. They practice recording their thinking in a variety of ways. Estimation, that requires students to consider where the decimal point goes—continues to build place value understanding in this lesson.

**Skills & Concepts**
- ★ Estimate the product of multiplying decimal numbers to determine a reasonable answer.
- ★ Multiply and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between multiplication and division (CCSS 5.NBT.7)
- ★ Relate the strategy to a written method and explain the reasoning used. (CCSS 5.NBT.7)

**Instructions for Using Models and Strategies**
1. Display the first word problem and ask students to read the problem, consider a reasonable estimate and record it in their journal. Invite them to pair share and then share their thinking with the class.

| Students | I think 112 divided by 10 would be 11.2 by its only 9 so it has to be a little less than that. I think its going to be more than 11.2… because you are only making 9 groups instead of 10. 9 × 10 is 90, and then you still have $22 left so maybe 2 more, $12? |

**You’ll Need**
- ★ Using Models & Strategies (page A11.57, run 1 for display and a class set)
- ★ Student Math Journals or Journal Page Grid (page A11.58 optional, run 1 copy for display plus additional copies as needed)
2. Work with your students to create a menu of quotients that will help them solve this division problem. Create a menu on a student journal page for the combinations that students suggest. Be sure the menu includes the multiplication facts below.

- $9 \times 1.00 = 9.00$
- $9 \times 2.00 = 18.00$
- $9 \times 10.00 = 90.00$
- $9 \times 20.00 = 180.00$

Then, give students a moment to re-think their estimates and share a few efficient strategies including sketches, menus and the partial quotients method. Be sure to discuss “where the decimal goes” in student strategies. Would $1.25$ make sense? Or $125.00$?

**Cedric** Since we said that $9 \times $10.00 = $90.00, I subtracted 90 from 112.50. Then I had $22.50 left so I used the menu to help me with $9 \times $2.00. I subtracted $18.00 from $22.50 and that only left $4.50. That’s half of $9.00, so I figured $.50¢ would work. Each week cost $12.50.

**Teacher** Did anyone think of it a different way?

**Ana Lucia** You could make an array, with 10, then 2 more and finally half of a dollar-to show 50¢.
3. Present Problem 2 and create a menu with students on a journal page. Be sure the menu includes these combinations:
   - \(8 \times 0.10 = 0.80\)
   - \(8 \times 1.00 = 8.00\)
   - \(8 \times 2.00 = 16.00\)
   - \(8 \times 10.00 = 80.00\)
   - \(8 \times 0.50 = 4.00\)

4. Ask students to look at the information on the menu and consider what a reasonable estimate would be for the quotient.

   Students
   
   Well, \$10 an hour would be \$80. That’s close enough.
   
   Well, I thought that too, but then you still have almost \$15 left. That’s almost enough for another \$2.
   
   I say close to \$12 an hour.

Give students time to solve the problem. After a few minutes, invite several students to share their thinking with the class. When students agree on the correct answer, record the decimal division equation \(94.00 ÷ 8 = \$11.75\).

   - Would an answer of \$117 per hour make sense? Would \$1.17 make sense?
   - Give students time to think, pair and share their understanding.

5. Continue with Problem 3. Invite students to create a menu and share what particular multiplication combinations are helpful to estimate a reasonable quotient. This particular situation, like many real world scenarios, does not require an exact answer. If none of your students bring that point up, you may want to. In this real world situation, an estimate, rather than an exact answer, is more appropriate. This problem will include a remainder.

3 Dan can travel about 526 miles on a full tank of gas. His car has a tank that holds about 14.60 gallons of gas. About how far can Dan travel on one gallon of gas?
Students  My parents get about 30 miles to the gallon, so I think a menu for 30 would be good—that’s my estimate too.
Well I thought about it like 15 gallons × 40 miles would be 600 miles, and that’s too high.
How come this problem keeps saying “about”?
Well, it’s not going to be an exact number when you are driving. It depends on the traffic and how fast you are going, I think.

6. Before sending students out to work independently, with a partner or in a small group with you, remind students to estimate a reasonable solution, make a menu, and then choose a division strategy that makes sense to them. They may use a quick sketch, partial quotients or the traditional algorithm. Have the students use one method to solve for the word problem and a second method to double-check their thinking. While students are working, monitor their thinking and invite them to convince you of the quotient, relating to the placement of the decimal.

INDEPENDENT WORKSHEET

Use Independent Worksheet 7 on pages A11.73 and A11.74 for more practice estimating and dividing whole numbers and decimals.
## Using Models & Strategies

### 1. If school lunches cost $112.50 per quarter—9 weeks. About how much would each week of lunches cost?

<table>
<thead>
<tr>
<th>× Menu for ____</th>
</tr>
</thead>
<tbody>
<tr>
<td>____ × ____ = ____</td>
</tr>
<tr>
<td>____ × ____ = ____</td>
</tr>
<tr>
<td>____ × ____ = ____</td>
</tr>
<tr>
<td>____ × ____ = ____</td>
</tr>
</tbody>
</table>

### 2. A fifth grader earned $94.00 gardening this month for a neighbor. If she worked 8 hours this month, then how much did she earn per hour?

### 3. Dan can travel about 511 miles on a full tank of gas. His car has a tank that holds about 14.60 gallons of gas. About how far can Dan travel on one gallon of gas?

### 4. Marcy joined the school track team and ran a total of 231.80 miles in practice over 61 days. About how many miles did she average per day?

### 5. A store owner had 7.11 lbs. of nuts left in the bin. If he put the nuts into 9 jars, how much will each jar of nuts weigh?

### 6. There are 2.54 centimeters in one inch. How many centimeters are in 38.10 inches?
Olympic Swimmers

For each problem, first estimate the answer and then solve the problem. Show your thinking using words, numbers, and/or labeled sketches.

1 In the 2012 Olympics, U.S. athlete Nathan Adrian finished the 100-meter freestyle swim in 47.52 seconds. If Adrian swam in a regular 25-meter pool, what would his time have been per lap?

Estimate __________________________
Answer __________________________

2 Dana Vollmer set a world record in the 100-meter butterfly finals in London. Her time was 55.98 seconds. If she swam in a 25-meter pool, what would Dana's time be per lap?

Estimate __________________________
Answer __________________________

3 Missy Franklin competed in seven Olympic swimming events and posted five gold medals in London. Her time in the 100-meter backstroke was 58.33 seconds. If Missy were swimming in a 25-meter pool, what would her time be per lap?

Estimate __________________________
Answer __________________________

(Continued on next page.)
4 Michael Phelps has 14 gold and 16 overall Olympic medals! In London, he won a gold medal for the 100-meter butterfly with a time of 51.21 seconds. If Michael were swimming in a 25-meter pool, what would his time be per lap?

Estimate __________________________  Answer __________________________

CHALLENGE

5 The men's 4 × 100 meter medley was won with a time of 3:29.35.

a If each of the four members of the team posted the same time, what would their individual times be?

Estimate __________________________  Answer __________________________

b If the men's swim team coach wanted to be sure the team was on track to win the gold medal, what times would each member have needed to post per 50 meter lap?

Estimate __________________________  Answer __________________________

Note  Did you know that Olympic length pools are actually 50 meters long?
Olympic Track Star

Solve each problem. Show your thinking using words, numbers, and/or labeled sketches.

1 Usain Bolt won 3 gold medals in the Track and Field events in the 2012 Olympics in London. His times are posted below.

<table>
<thead>
<tr>
<th>Race</th>
<th>Time in Seconds</th>
</tr>
</thead>
<tbody>
<tr>
<td>Men’s 100 meter</td>
<td>9.63</td>
</tr>
<tr>
<td>Men’s 200 meter</td>
<td>19.32</td>
</tr>
<tr>
<td>Men’s 4 x 100 meter relay</td>
<td>36.84</td>
</tr>
</tbody>
</table>

a Bolt ran the 200 meters in 19.32 seconds. If he ran 100 meters at that pace, what would his 100 meter time be?

b For 100 meters, what’s the difference between Usain’s 100 meter pace and his 200 meter pace?

c Four Jamaican runners ran the men’s 4 × 100 meter relay with a time of 36.84. If each ran the same speed, what would one runner’s time have been?

d If the relay runners could run as fast as Bolt did in his individual 100 meter race, would their relay time have been faster or slower? By how much?

(Continued on next page.)
2 Divide each number. Show your work.

<table>
<thead>
<tr>
<th>Number</th>
<th>Divided by 10</th>
<th>Divided by 100</th>
</tr>
</thead>
<tbody>
<tr>
<td>9.6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>16.08</td>
<td></td>
<td></td>
</tr>
<tr>
<td>132.22</td>
<td></td>
<td></td>
</tr>
<tr>
<td>78.2</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

3 Compare what happens to the quotient when you divide by 10 and by 100.

4 Compare what happens to the quotient when you divide by 10 and by 20.

5 Kary and Val were solving the following problem: $12.55 ÷ 5. Kary wrote $25.10 as her answer. Val wrote $2.51. Who is right? How do you know?
GRADE 5 – UNIT 5

CCSS SUPPLEMENT ACTIVITIES & INDEPENDENT WORKSHEETS

Set A12: Number & Operations: Dividing Fractions & Whole Numbers
  Activity 1: Dividing Fractions & Whole Numbers Pre-Assessment ............ A12.1

Set A9: Number & Operations: Multiplying Fractions
  Activity 1: Geoboard Perimeters ............................................ A9.1
  Activity 2: Fraction Multiplication Story Problems .......................... A9.9
  Activity 3: Using the Area Model for Multiplying Fractions ............... A9.17
  Activity 4: Generalizations About Multiplying Fractions .................. A9.23
  Ind. Worksheet 1: Sharing & Grouping; Multiplying & Dividing .......... A9.49
  Activity 5: Target 1: Fractions ............................................. A9.31
  Ind. Worksheet 2: More Fraction Multiplication ........................... A9.51
  Activity 6: Multiplying Domino Fractions .................................. A9.37
  Ind. Worksheet 3: Fraction Stories ......................................... A9.53
  Activity 7: Area Word Problems with Mixed Numbers ..................... A9.45
  Ind. Worksheet 4: Using Strategies to Multiply Fractions with Mixed Numbers ...... A9.55

Set A12: Number & Operations: Dividing Fractions & Whole Numbers
  Activity 2: Reviewing the Sharing & Grouping Interpretations of Division .... A12.7

Set A9: Number & Operations: Multiplying Fractions
  Ind. Worksheet 5: Domino Multiplication .................................. A9.57

Set A12: Number & Operations: Dividing Fractions & Whole Numbers
  Activity 3: Grouping Stories ................................................. A12.17
  Ind. Worksheet 1: Sharing & Grouping; Multiplying & Dividing .......... A12.75
  Activity 4: Dividing a Whole Number by a Fraction ....................... A12.27
  Ind. Worksheet 2: Operating with Fractions & Whole Numbers .......... A12.77
  Activity 5: Sharing Stories .................................................. A12.37
  Activity 6: Dividing a Fraction by a Whole Number ....................... A12.49
  Ind. Worksheet 3: More Fractions & Whole Numbers ..................... A12.79
  Activity 7: The Division Poster Project .................................... A12.59
  Activity 8: Dividing Fractions & Whole Numbers Post-Assessment ...... A12.65
Set A12 ★ Activity 1

Dividing Fractions & Whole Numbers Pre-Assessment

Overview
This pre-assessment launches a set of activities that deal with division of fractions by whole numbers and whole numbers by fractions. It is strongly recommended that teachers read the background information below and conduct the pre-assessment before teaching Activities 2–7 in this set. During Activity 8, students will take an expanded version of today’s assessment.

Skills & Concepts
★ Multiply a whole number by a fraction (CCSS 5.NF.4a)
★ Solve story problems involving multiplying a whole number by a fraction (CCSS 5.NF.4a)
★ Divide a unit fraction by a whole number (CCSS 5.NF.7a)
★ Divide a whole number by a unit fraction (CCSS 5.NF.7b)
★ Write story problems involving division of a unit fraction by a whole number (CCSS 5.NF.7b)
★ Solve story problems involving division of a unit fraction by a whole number (CCSS 5.NF.7c)
★ Solve story problems involving division of a whole number by a unit fraction (CCSS 5.NF.7c)

You’ll need
★ Operating with Fractions & Whole Numbers Pre-Assessment (pages A12.3–A12.5, run a class set plus a copy for display)
★ Operating with Fractions & Whole Numbers Pre-Assessment Class Checklist (optional, page A12.6, run 1 or 2 copies)

Instructions for Dividing Fractions & Whole Numbers Pre-Assessment
1. Explain to students that over the next couple of weeks, the class will study division of fractions and whole numbers. Today they’ll take a pre-assessment that will provide you information about what they already know and what they still need to learn regarding the skills and concepts involved. Explain that in about two weeks, they will take a similar assessment, at which time they will have additional ways to handle problems that may seem challenging today.

2. Give students each a copy of the pre-assessment. Ask them to write their name and the date at the top of each page. Read and review the problems together and have students circle the “doing” words as you go.
Activity 1  Dividing Fractions & Whole Numbers Pre-Assessment (cont.)

3. Before students start to work, be sure they understand they have to show their work and/or explain their thinking for problems 1 and 2; the answers alone will not be adequate. Remind them to write a story problem to match the expression in problem 3, to solve the problem and write the answer. Remind students of the difference between an expression (12 ÷ 2) and an equation (12 ÷ 2 = 6). An expression is a mathematical phrase without an equal sign. An equation completes the expression with a solution after an equal sign. You might list examples for the students on the board (e.g. expressions: 3 + 4, 27 + 9, equations 3 + 4 = 7, 27 + 9 = 36.) Also, alert them to the fact that this assessment includes multiplication and division situations because it’s important to determine which operation is called for in a given problem.

Let students know that you can’t explain the tasks to them, but you will reread any of the problems to them if needed during the assessment period. Although they may not be sure how to solve some of the problems, encourage them to attempt each one. Partial solutions are fine, and if they are unable to answer a particular question or solve a particular problem they can write, “I don’t know yet.” You might also have them underline any words they don’t understand.

4. Students will complete a similar assessment in Set A12, Activity 8, at which time a scoring guide will be included for your use. We recommend that you use the results of today’s pre-assessment to help guide your instruction as you teach this set of activities. To help, you can use the Dividing Fractions & Whole Numbers Pre-Assessment Class Checklist on page A12.6 if you like. By compiling results for your entire class, you can get a sense of the areas in which the class as a whole may need extra support.
1. Solve each of the three story problems below. For each problem:
   • Write an expression to represent the problem.
   • Use numbers, visual models, labels, and/or words to solve the problem.
   • Complete the sentence below with your solution to the problem.

   a. Shelly made cupcakes and now she wants to frost them. She has 4 cups of frosting. It takes $\frac{1}{3}$ a cup of frosting for each cupcake. How many cupcakes can she frost?

   Expression: ______________________________________________________________

   Shelly can frost _____ cupcakes.

(Continued on next page.)
Jake and his dad are making flags for a scouting project. They are going to make 6 flags and need $\frac{2}{3}$ a yard of cloth for each flag. How many yards of cloth will they need in all?

Expression: _______________________________

Jake and his dad will need _____ yards of cloth in all.

C. There is $\frac{1}{2}$ a pan of brownies left. Four children are going to share it equally. What fraction of the whole pan of brownies will each child get?

Expression: _______________________________

Each child will get _____ of the whole pan of brownies.

(Continued on next page.)
2 Cory says that $6 \div \frac{1}{2}$ means the same thing as $\frac{1}{2}$ of 6, so the answer is 3. Do you agree with him? Why or why not? Use numbers, labeled models, and/or words to explain your thinking.

3 Write a story problem to represent the expression in the box below. Then solve your own problem. Show your work with labeled models, numbers, and/or words, and write the answer on the line provided.

$$\frac{1}{2} \div 3$$

My story problem:

My work:

The answer to my problem is ________.
Operating with Fractions & Whole Numbers Pre-Assessment

<table>
<thead>
<tr>
<th>Item and Correct Answer</th>
<th>CCSS</th>
<th>Student Name</th>
</tr>
</thead>
<tbody>
<tr>
<td>1a Write an expression to represent a story problem that involves dividing a whole number by a unit fraction. Solve the problem; show work. Expression: (4 \div \frac{1}{3}) Answer: 12 cupcakes; student work will vary</td>
<td>5.NF.7b, 5.NF.7c</td>
<td></td>
</tr>
<tr>
<td>1b Write an expression to represent a story problem that involves multiplying a whole number by a fraction. Solve the problem; show work. Expression: (6 \times \frac{2}{3}) OR (\frac{2}{3} \times 6) Answer: 4 yards of cloth; student work will vary</td>
<td>5.NF.4a</td>
<td></td>
</tr>
<tr>
<td>1c Write an expression to represent a story problem that involves dividing a unit fraction by a whole number. Solve the problem; show work. Expression: (\frac{1}{2} \div 4) Answer: (\frac{1}{8}) the pan of brownies; student work will vary</td>
<td>5.NF.7a, 5.NF.7c</td>
<td></td>
</tr>
<tr>
<td>2 Interpret division of a whole number by a unit fraction. Answer: No; students’ explanations will vary. Example: I don’t agree because (6 \div \frac{1}{2}) is not the same thing as half of 6. It means how many halves in 6. Since there are 2 halves in 1, there are 12 halves in 6.</td>
<td>5.NF.7b</td>
<td></td>
</tr>
<tr>
<td>3 Write and solve a story problem to represent the expression (\frac{1}{2} \div 3); show work. Answer: (\frac{1}{6}) Students’ story problems and work will vary. Example: I had half a sandwich. I gave it to my 3 friends to share equally. How much of the sandwich did each friend get?</td>
<td>5.NF.7a</td>
<td></td>
</tr>
</tbody>
</table>
Geoboard Perimeters

Overview
In preparation for using the area model to multiply one fraction by another, students investigate the perimeter of the largest square that can be formed on the geoboard, as well as the perimeters of smaller regions on the geoboard.

Skills & Concepts
★ add fractions with unlike denominators
★ find the perimeter of regions with an area smaller than 1

You’ll need
★ Rectangle Review (page A9.6, run 1 for display)
★ Geoboard Perimeters (page A9.7, run 1 for display)
★ More Geoboard Perimeters (page A9.8, run a double-sided class set, plus a few extra)
★ geoboard and geobands (class set plus 1 for display)
★ pens
★ 2–3 blank transparencies
★ a piece of paper to mask portions of the display
★ 5 ⅞” × ⅛” strips of red construction paper (10–12 per student)
★ tile and red linear units available as needed
★ pencils and scissors

Note  When you represent the symbolic form for a fraction, please use a horizontal bar.

Instructions for Geoboard Perimeters
1. Open the activity by explaining to the class that you are going to start a series of lessons on multiplying fractions. To get started, you are going to review the area model for multiplication. Then display the Rectangle Review master. Review the information together, and ask students to pair-share responses to the questions:
   • What is the area of the rectangle on the display?
   • What information do you need in order to determine the area of the rectangle?

2. Have a few volunteers share their thinking with the class. As the discussion proceeds, guide students to review the connection between perimeter, area, and multiplication.

   Students  We think it’s about 28 square inches.
   We said it could be maybe be about 150 square centimeters.
   We can’t tell, because we don’t know how long the sides are.
   We don’t even know if they’re in inches or centimeters.

   Teacher  Why do you need to know the side lengths to find the area of the rectangle?

   Students  Because you get area by multiplying length times width.
   You need to know how many squares will fit into the rectangle. Like, if we know that 7 squares fit
across the top, and 4 squares fit along the side, we would know the area is 4 times 7, and that’s 28. But it depends on the size of the squares. If they’re little, like square centimeters, the area could be more than 100.

3. After some discussion, have a volunteer come up to the display and measure the side lengths of the rectangle in inches. Then work with input from the class to label the rectangle and summarize students’ comments on the display.

4. Next, display the top portion of the Geoboard Perimeters master as helpers give students each a geoboard and some geobands. Read the information on the display together and ask students to replicate the square on their own geoboard. If the area of that square is 1 unit, what is the length of each side, and what is the perimeter of the square? Give students a minute to pair-share ideas, and then call for and record their answers.

**Teacher** Now that you’ve had a minute to think about the question, let’s record your answers here on the whiteboard. What did you decide?
Activity 1 Geoboard Perimeters (cont.)

**Students** We don’t agree with Jason. We think the perimeter of that square is 16. That’s what we got too. We agree with Jason. We think the perimeter is 4.

5. After you have recorded students’ answers, invite individuals or student pairs to the display to demonstrate their thinking. Set a blank acetate on top of your master and then re-position it as needed, so that several different students can mark on it to show how they determined the perimeter of the square in question.

**Teacher** Any different ideas? No? Who’d like to convince us of their reasoning? You can mark on the display to show what you did to get your answer.

**Jon** We said it was 16 instead of 4. We started in the corner of the board and just counted the pegs all the way around. It came out to 16.

![Diagram of a geoboard with pegs]

**Ariel** We did kind of the same thing as Jon and Omid, but we looked at the spaces instead of the pegs. It looked like each side of the square was 4, and we know that 4 \times 4 is 16, so we said the perimeter of the square is 16.

![Diagram of a geoboard with pegs]

**Gabe** We think the perimeter is 4. We said if the area of the whole square is 1, then each side must be 1. So that means the perimeter of the square is 4, like this: 1, 2, 3, 4.

![Diagram of a geoboard with pegs]

**Jasmine** We agree with Gabe and Raven. See, if each of the little squares was worth 1, then the perimeter would be 16, but the big square is worth 1, so each of the sides must be 1.

6. When students have had adequate time to discuss and debate the perimeter of the largest square, build the square on your own geoboard at the display and show one of the strips of red construction paper you have cut, first holding it up for all to see, and then setting it into the space between the edge and the pegs of the board. Then invite students’ comments.

**Teacher** I cut some strips for us to use in considering the perimeter of this square. What do you think?
Students Those are like the little red pieces we use with the tiles sometimes. It's like a giant red piece. But those little red pieces are worth 1, so this one must be worth 4.

Teacher How are you thinking about that?

Kamil Well, it goes along 4 spaces on the geoboard, so it must be worth 4.

Hanako But that's what we were trying to tell you before. That square has an area of 1. It's like 1 giant tile, and that strip is like 1 giant red piece.

7. Confirm the fact that the red strips you have cut are each worth 1 linear unit. That being the case, what is the perimeter of the largest square on the geoboard? (4 linear units)

8. Now display the middle portion of the master, which establishes that the perimeter of the largest square is 4 linear units and asks students to determine the perimeter of several different regions on the geoboard.

9. Work with the class to determine the perimeter of Region B. Ask students to remove the large square from their board and build just Region B, as you place a handful of red construction paper strips at each table or cluster of desks. Give students a few minutes to experiment with their strips as they consider the perimeter of this region. Let them know that it is fine to fold and cut the strips if that helps them think about the length of each side of Region B. Then invite 2 or 3 individuals or pairs to the display to share their thinking. Ask them to work with a board and strips so their classmates can see what they are talking about as they explain.

Theo We were pretty stuck at first, but we kept looking at the strips and the rectangle on our board. Then we realized that if you fold one of the strips in half, it fits along the top of the rectangle. Then we knew that the 2 long sides were each worth \(\frac{1}{2}\).

Ichiro We found out that the small sides are each worth \(\frac{1}{4}\) of a linear unit. If you fold one of those strips in half and then in half again, you get fourths. If you cut them up, they fit right along the short sides of the rectangle, like this.
Students who determine and record the perimeters of all 5 regions quickly and easily can be asked to build at least two figures (other than any of the regions they've already investigated) that have a perimeter of 2 linear units, two that have a perimeter of $2\frac{1}{2}$ linear units, two with $P = 3$ linear units, and two with $P = 3\frac{1}{2}$ linear units. Each discovery should be recorded the same way the first 5 regions have been, using the last box on the record sheet, as well as the back of the sheet and a second sheet if necessary.
Rectangle Review

What is the area of this rectangle?

What information do you need before you can answer the question?

How are perimeter, area and multiplication related?
Jason says that the perimeter of this square is 4 linear units. Do you agree with him? Why or why not?

Area = 1 Square Unit

If the biggest square on the geoboard has a perimeter of 4 linear units, what is the perimeter of each lettered region?

Perimeter = 4 Linear Units
More Geoboard Perimeters

A

B

C

D

E

P = ___________ linear units

P = ___________ linear units

P = ___________ linear units

P = ___________ linear units

P = ___________ linear units
Fraction Multiplication Story Problems

Overview
During this session, students solve several different story problems designed to help them think sensibly about multiplying one fraction by another. Although the expression \(\frac{1}{4} \times \frac{1}{2}\) may not carry much meaning for most fifth graders, many students can consider the idea of “a fourth of a half”, especially in the context of a story problem supported by visual models. Today’s sense-making activities lay the groundwork for using the area model to picture and solve fraction multiplication combinations in the next activity.

Skills & Concepts
★ find the product of two unit fractions with small denominators using an area model
★ estimate the results of operations performed on fractions and use the estimate to determine the reasonableness of the final answer
★ explain the relationship of the product relative to the factors when multiplying fractions

Instructions for Fraction Multiplication Story Problems
1. Open today’s activity by placing the top portion of The Brownie Problem on display as students gather the materials they’ll need: geoboards and bands, journals and pencils.

2. Read the problem at the top of the display master together, along with the instructions. Then ask students to think privately about the situation, and record an estimate in their journal, along with an explanation. Encourage students who seem puzzled or confused to think in terms of familiar benchmarks. Did Maribel eat more or less than half a pan? Did she eat more or less than a quarter of the pan? Why?

3. After they have had a few minutes to write, ask students to pair-share their estimates and explanations and then work in pairs to build the situation on a geoboard. Students will have different ideas...
about how to do this, and you may see some misconceptions as you circulate, but give them a few minutes to wrestle with the problem and record answers in their journals.

Ask students who finish before their classmates to determine what part of a pan of brownies Maribel would have eaten if there had been three-fourths of a pan left instead of half ($\frac{1}{4}$ of $\frac{3}{4}$ is $\frac{3}{16}$).

4. When most students have either arrived at an answer or done as much as they can with the problem, record all solutions on the whiteboard and then invite 2–3 individuals or pairs to the display to share their thinking. Have them bring their geoboards with them so their classmates can see how they built the situation and found their way to an answer.

**Teacher** Now that you’ve had a little while to investigate this problem, let’s share our answers and then have a few volunteers show us how they modeled the situation on their geoboards. What part of the pan of brownies did Maribel eat?

**Students** We said she ate $\frac{1}{4}$ of the pan.
We got $\frac{1}{8}$ of the pan.
We got $\frac{2}{16}$.

**Teacher** Who’d like to share their strategy for building this problem on the geoboard?

**Rian** We said if the geoboard is the whole pan of brownies, we only need half because that’s all that was left. So we made a rectangle on half the board, like this.

![Geoboard diagram](image)

**Beth** Then we had to find a fourth of that because the problem said she ate a fourth of what was left. So we divided the half into 4 parts, like this. Then we had to think about how big one of those little parts was. We could see that each of the little pieces took up 2 squares, and we know that each square is $\frac{1}{16}$, so we said that she ate $\frac{2}{16}$.

![Geoboard diagram](image)

**Darius** We built ours the other way, like this. We could see that there would be 8 parts like that if you filled the whole pan, so one of them is $\frac{1}{8}$ of the pan.

![Geoboard diagram](image)
**Teacher** Beth and Rian say the answer is $\frac{2}{16}$. Darius and Javier say it’s $\frac{1}{8}$. Is there any connection between the two?

**Students** Sure! They’re the same thing. It’s 2 different names for the same fraction. But I don’t get it. The story says Maribel ate $\frac{1}{4}$ of what was left. And each one of those pieces is a fourth, so why are you saying that she ate $\frac{1}{8}$ or $\frac{2}{16}$? We thought the answer was $\frac{1}{4}$.

**Teacher** Can anyone respond to Josie’s question?

**Morgan** Well, you’re right. But each of those pieces on the geoboard up there is a fourth of a half a pan, not a fourth of a whole pan. It is okay if I move your geobands, Darius?

See, if you look at just the piece she ate, it’s $\frac{1}{8}$ of the board, like Region B, remember? I brought my board up too, and that shows a fourth of the board so you can see the difference. Maribel only got a fourth of what was left, not a fourth of the whole pan.

**Josie** I think I see, but this is kind of confusing.

5. After several students have shared their strategies and there is general consensus that the answer is $\frac{1}{8}$, work with student input to create a sketch of the situation on the grid in the middle section of the display master, along with a written description of what happened.

6. Then explain that the expression a mathematician would use to represent the situation is $\frac{1}{4} \times \frac{1}{2}$, which is read as, “one fourth of one half”. Record the full equation below the grid at the display. (The expression $\frac{1}{4} \times \frac{1}{2}$ can also be read as, “one fourth times one half”, but we find that if we encourage our students to read it the other way, it taps into their sense-making abilities much more effectively.)
7. Ask students to make a sketch similar to the one on the overhead in their journal, shading in with a colored pencil the part of the pan of brownies that Maribel ate. Then ask them to write a description of what happened, adding any other observations they have, and an equation to match.

8. Next, display the Fraction Multiplication Story Problems master. Review the instructions at the top with the class.

9. There are several ways you might handle the remainder of the activity, depending on the strengths and needs of your class.

- Do one of the five problems as a group. You can re-use the middle portion of the Brownie Problem display master to record a sketch, written description, and equation to match the situation. After completing one of the problems, have students work in pairs to do the rest, each individual responsible for completing all the steps in his or her journal.
- Allow those students who feel ready to work independently to do so, while you work with the others, going through as many of the problems as time allows.
- Work through as many of the problems at the display as time allows with the entire class.

Whether you choose to have student pairs work independently on some or all of these problems or keep the entire class together, ask students to follow each of the steps outlined on the display master. Writing a description of what happened, including the answer, and an equation to match may be the most challenging part for some of your students. However, if they come away from today’s experiences understanding that \( \frac{1}{4} \times \frac{1}{2} \) means half of a fourth, able to picture such a situation and think about it sensibly, they’ll be well on their way to developing the insights they need to understand multiplication of fractions.
The Brownie Problem

When Maribel got home from school yesterday, she went into the kitchen to get a snack. There was $\frac{1}{2}$ of a pan of brownies on the counter. Maribel ate $\frac{1}{4}$ of what was left. What part of the pan of brownies did Maribel eat?

1. Record an estimate in your journal and then write a sentence or two to explain it. What part of the pan of brownies do you think Maribel ate? Why?

2. Build the situation on your geoboard and record the answer next to your estimate. (Pretend that the whole board is 1 whole pan of brownies.)

Description of what happened:

Equation: __________________________
Fraction Multiplication Story Problems

Here are 5 more fraction multiplication story problems. For each one that you and your partner solve, do the following:
• Write the problem number in your journal.
• Record an estimate. What do you think the answer will be and why?
• Build the situation on your geoboard and then make a labeled sketch of it in your journal.
• Write a description of what happened that includes the answer.
• Write a multiplication equation to match.

1 When Max got home from school yesterday, he went into the kitchen to get a snack. There was $\frac{3}{4}$ of a pan of brownies on the counter. Max ate $\frac{1}{4}$ of what was left. What part of the pan of brownies did Max eat?

2 Brittany’s mom had a big garden last summer. She planted corn and tomatoes in $\frac{1}{2}$ of the garden. She planted lettuce in $\frac{1}{4}$ of the garden. She used $\frac{1}{2}$ of the last $\frac{1}{4}$ of the garden for flowers. How much of the garden did she use for flowers?

3 Dontrelle was getting a new rug for his bedroom. His dad said that the rug would cover $\frac{3}{4}$ of half the floor. How much of the floor did the rug cover?

4 Maria had $\frac{7}{8}$ of a box of candy left from her birthday. She gave half of what she had left to her little sister. How much of the box of candy did her little sister get?

5 Marco had $\frac{3}{8}$ of a pizza left from dinner the night before. He gave half of what he had left to his friend. How much of a pizza did his friend get?
Journal Page Grid
Using the Area Model for Multiplying Fractions

Overview
Students use the area model to multiply fractions, as they build rectangles on their geoboards with fractional dimensions and find the areas. Then students write their own story problems to accompany one or more of the fraction multiplication combinations with which they have worked today.

Skills & Concepts
★ find the product of two unit fractions with small denominators using an area model
★ estimate the results of operations performed on fractions and use the estimate to determine the reasonableness of the final answer
★ explain the relationship of the product relative to the factors when multiplying fractions

You’ll need
★ The Brownie Problem from Activity 2 (page A9.19 run 1 copy for display, see Advance Preparation)
★ Student Math Journals and/or Journal Page Grid (page A9.22, run 1 copy for display and additional copies as needed)
★ geoboard and geobands for display
★ pens in several colors, including red
★ a piece of paper to mask portions of the display
★ 5 3/4” × 1/4” strips of red construction paper (available to students as needed)
★ regular and colored pencils (each student will need red and one other color)

Advance Preparation To start this session, you’ll need to have The Brownie Problem display master from Activity 2 filled in with the solution to the original problem. If you used this display master to model responses to some of the other story problems during the previous activity, erase the work and enter a sketch, written description, and number sentence for the original problem.

Note When you represent the symbolic form for a fraction, please use a horizontal bar.

Instructions for Using the Area Model for Multiplying Fractions
1. Open today’s session by placing a small pile of the 5 3/4” × 1/4” red construction paper strips on each table or cluster of desks so students will have easy access to them. Then show the top portion of The Brownie Problem display master from Activity 2 as students get out their journals, geoboards, and geobands.

2. Read the problem with the class and ask students to pair-share the responses they recorded in their journals during the previous activity. Then ask each student to build on his or her geoboard just the part...
of the pan of brownies Maribel got to eat. What are the dimensions and the area of this rectangle? Give students a minute to pair share, using some of the red construction paper strips to help figure it out if they need to.

3. Then show the entire display master and invite a student volunteer to the display to sketch the rectangle on the geoboard at the bottom of the sheet, labeling the dimensions and area with input from classmates.

4. Now record the following expression on the display, as students do so in their journals.
   - $\frac{1}{2} \times \frac{1}{2}$

Ask students to read the expression, using the same language they did during the previous activity: “One half of one half” or “Half of a half”. Give them a minute to record an estimate, with the understanding that they may be called upon to explain their thinking. Then call on a student or two to share and explain their estimates.

   **Jasmine** I know it’s going to be less than a half, because it’s only half of a half, so it can’t be the whole thing.

   **Javier** I said it was going to be $\frac{1}{4}$, because if you cut $\frac{1}{2}$ in half, you get $\frac{1}{4}$.

5. Next, ask students to build a square with dimensions $\frac{1}{2} \times \frac{1}{2}$ on their geoboards, working together to share and compare ideas. If they are not sure how to build the figure, encourage them to use the red construction paper linear strips, remembering that each strip has been assigned a value of 1 linear unit.
Activity 3  Using the Area Model for Multiplying Fractions (cont.)

*Darius*  Okay, half of a half. I know it’s going to be a fourth, but I’m not sure how to show it on the geoboard. How do you make something that’s a half by a half?

*Armin*  We can use those red strips, remember? They’re like giant linear units, so we can fold one in half and put it next to the geoboard to help.

Here’s a half strip, right? Then the other side of the square is also a half. So now we can make the square with a rubber band.

*Darius*  Oh yeah—it’s all coming back to me. And look! A square that’s half by a half really does turn out to be one-fourth of the board.

*Kamela*  That’s because half of a half is a fourth, just like we thought it would be.

6. When most have completed this task, ask a volunteer to build the figure at the display, explaining her thinking as she does so. Then work with student input to make a sketch of the problem on the Journal Page Grid display master.

*Teacher*  How can we make a sketch of this problem on the kind of grid paper you have in your journals?

*Justin*  First you need to outline a geoboard, like a 4 × 4 square.

*Teacher*  And then?

*Raven*  Then you need to draw in the dimensions—how long each side of the square is going to be.

*Teacher*  Raven, why don’t you come up and do that for us. Go ahead and use the red pen to show the dimensions. That way they’ll match the color of our linear strips.
Teacher  Thanks, Raven. What should we do now, class?

Jade  Now just draw in the rectangle and put a label that shows its area, like this. Let’s color it in so it shows up better.

7. Once a sketch of the combination and the solution has been created at the display, have students replicate the sketch in their journal, using red colored pencil to show the dimensions and a second color to shade in the square that results.

8. Repeat steps 4–7 with the following combinations:
   •  \( \frac{1}{4} \times \frac{1}{4} \)
   •  \( \frac{1}{4} \times \frac{3}{4} \)
   •  \( \frac{1}{4} \times 1 \)
   •  \( \frac{1}{2} \times \frac{3}{4} \)

In each case, have students complete the following steps:
   •  Read the expression after you’ve written it on the board, using the word of instead of times, i.e., “one fourth of one fourth” or “one fourth of three fourths.”
   •  Record the expression in their journal along with an estimate of the answer.
   •  Build the combination on their geoboard, working with the students nearest them to share and compare ideas and results. Invite at least one volunteer to the display to share his thinking, using his own board. If there is confusion or debate, you may want to have several students share their thinking with the class. Be sure to bring misconceptions into the open so everyone benefits.
   •  Record both a sketch and the answer in their journal.
9. Conclude the activity by asking students to write a story problem to accompany at least one of the fraction multiplication problems they have done today. Here are several examples of the kinds of story problems we’ve seen fifth graders write in response to this assignment.

- My little brother is always coming into my room and bothering me, so finally I got some tape and marked off a fourth of a fourth of the room for him to play in. What part of the room did he get? \( \frac{1}{4} \times \frac{1}{4} = \frac{1}{16} \)

- My dad said we could set up our volleyball net in the back half of our yard. When we did, it took up \( \frac{3}{4} \) of the space. How much of the yard did it fill? \( \frac{3}{4} \times \frac{1}{2} = \frac{3}{8} \).

- I had \( \frac{3}{4} \) of a candy bar left. I gave \( \frac{1}{4} \) of that to my best friend. How much of my candy bar did she get? \( \frac{1}{4} \times \frac{3}{4} = \frac{3}{16} \)

**Extensions**

- Post a display in the hall that shows each multiplication combination, accompanied by a sketch and the student-written story problems that match.

- If there is another fifth grade class in your school using Bridges, have the two classes trade story problems. Each student in your room can solve one of the problems written by a student in the other class, and send back a very complete and detailed record of his or her solution and strategy. (Our students really enjoy seeing how other fifth-graders solve their problems.)
Journal Page Grid
Generalizations About Multiplying Fractions

Overview
Students sketch and solve a variety of fraction multiplication combinations on grid paper. The teacher then guides them to the generalization that the product of any two fractions can be found by multiplying their numerators and then multiplying their denominators.

Skills & Concepts
★ find the product of two unit fractions with small denominators using an area model
★ multiply fractions using the standard algorithm
★ explain the relationship of the product relative to the factors when multiplying fractions

You’ll need
★ Fractions to Multiply (page A9.28, run 1 copy for display)
★ Multiplying Fractions (pages A9.29 and A9.30, run 1 copy of each for display, and a class set)
★ Student Math Journals or Journal Page Grid (pages A9.22 optional, run copies as needed)
★ red, blue and black pens for the display
★ paper to mask portions of the display
★ colored pencils

Note When you represent the symbolic form for a fraction, please use a horizontal bar.

Instructions for Generalizations about Multiplying Fractions

1. Have students open their math journals to the work they did during the previous session. Ask them to pair-share any observations they can make so far about multiplying fractions. After a minute or two, ask volunteers to share their thinking with the class.

Students When you multiply a fraction by 1, it stays the same, like 1/4 times 1 is just 1/4.
It seems like when you multiply one fraction by another, the answer is always smaller than what you started with.
Yeah, like 1/4 times 1/4 is 1/16. It’s kind of weird, because usually when you multiply, you get a bigger answer.
It makes sense if you remember that 1/4 × 1/4 is really one-fourth of a fourth.
I think you can just get the answers by multiplying across.
Teacher  Multiplying across? How are you thinking about that, Brianna?

Brianna  Can I show at the board? Look, we know that a fourth of a fourth is a sixteenth, right?
So just multiply 1 times 1, and you get 1, and 4 times 4 and you get 16. Or 1/4 times 3/4 is 3/16. We showed that on our boards yesterday. But look what happens when I write the numbers and multiply across. 1 times 3 is 3, and 4 times 4 is 16!

\[
\begin{array}{c}
\frac{1 \times 1}{4 \times 4} = \frac{1}{16} \\
\frac{1 \times 3}{4 \times 4} = \frac{3}{16}
\end{array}
\]

Teacher  So, you're saying that if you multiply the numerators, and then multiply the denominators, you get the same answers we did when we built and sketched the combinations? That's an interesting observation. Do you think it will work every time? Let's keep the idea in mind as we look at some more problems today.

2. Explain that you have some more fraction multiplication problems for the class today. Then show the first problem on the Fractions to Multiply display master. Read the first problem with the students, and work with their input to record the phrase that will help them think sensibly about the combination. Have students pair-share estimates, and ask several volunteers to share their thinking with the class.

Students  We know it's going to be less than 3/5 because it's only half of that.
We think maybe the answer is going to be 3/10 because half of one fifth is one tenth, so maybe half of three fifths would be 3/10.
If you use my idea about multiplying across, you get 3/10.

3. Work with input from the class to frame a rectangle on the grid that is 1/2 by 3/5. Then shade in the resulting region, and ask students to identify the area of the rectangle relative to the whole grid.

Students  So, the answer is 3/5, right?
That doesn't make sense! It should only be half of 3/5.
But there are 3 boxes colored in and 5 in the row, so it's 3/5.
Wait a minute! There are 10 boxes in the whole grid. We marked half on one side and $\frac{3}{5}$ along the top, and the part we colored in is $\frac{3}{10}$.

4. When there is general agreement that the answer is $\frac{3}{10}$, record it on the display. Then show the next combination. Read it with the class and write a verbal “translation” below the problem. Have students open their journals to the next available page, record the combination, and outline a $3 \times 4$ rectangle. Give them a minute or two to solve the problem, sharing and comparing their work with others nearby as they work. Then invite a volunteer to the display to share his or her thinking with the class.

Sergio $\frac{6}{12}$ is half of the rectangle. See there are 6 white squares and 6 that are colored in. $6 + 6 = 12$

5. Repeat step 4 with the last combination on the display, $\frac{4}{5} \times \frac{2}{3}$. This time, however, students will have to decide what size rectangle to outline before they model the combination.

Teacher Can someone come up and mark a dimension of $\frac{4}{5}$ along the side of this grid? Maria?

Maria Sure! Just mark 4 down the side, like this.

Teacher Do you all agree that this shows $\frac{4}{5}$ of the side of the grid? Talk with the person next to you for a minute, and then let’s hear what you think.

Students Yep, we agree. It’s 4 down the side.
We don’t agree. There are 7 squares down the side. If you mark 4 of them, it’s like $\frac{4}{7}$, not $\frac{4}{5}$.
I think you can’t use that whole grid. You have to make one where you can show fifths on one side and thirds on the other.
6. After some discussion, work with input from students to outline a rectangle that will work for this combination. Then mark the dimensions, shade in the region that results, and record the answer at the display, as students do so in their journals.

7. Give students each a copy of the Multiplying Fractions sheets and display the master for everyone to see. Review and discuss the tasks with the class. Give students the option of working on these sheets independently, in pairs, or with you.
Activity 4  Generalizations About Multiplying Fractions (cont.)

8. After students have completed the worksheets, discuss the fourth problem with the group, and guide them to the generalization that the product of two fractions is found by multiplying the numerators and then multiplying the denominators. Also, ask students to explain why the product of two fractions is smaller than either of the factors.

   **Students**  Multiplying two fractions is like finding a fraction of a fraction. Half of a half has to be smaller than a half.
   Or, like if you find a fourth of a half, it’s only an eighth.
   It goes the other way, too. A half of a fourth is an eighth.

**INDEPENDENT WORKSHEETS**

See Set A9 Independent Worksheets 1–3 on pages A9.49–A9.53 for more practice with multiplying fractions.
<table>
<thead>
<tr>
<th>Fractions to Multiply</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{1}{2} \times \frac{3}{5} = )</td>
</tr>
</tbody>
</table>
| [Diagram of 1/2 of 3/5 shaded]
| \( \frac{2}{3} \times \frac{3}{4} = \) |
| [Diagram of 2/3 of 3/4 shaded]
| \( \frac{4}{5} \times \frac{2}{3} = \) |
| [Diagram of 4/5 of 2/3 shaded]|
Multiplying Fractions  page 1 of 2

1 Each of the pictures below shows the results of multiplying one fraction by another. Label each of the shaded regions with its dimensions and area. Then write a multiplication equation to match

![Example](example.png)

2 Fill in the chart to solve each of the problems below.

<table>
<thead>
<tr>
<th>Multiplication Equation</th>
<th>Word to Match</th>
<th>Labeled Sketch</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{2}{3} \times \frac{2}{3} = )</td>
<td>two-thirds of two-thirds</td>
<td><img src="labeled_sketch.png" alt="Labeled Sketch" /></td>
<td>( \frac{4}{9} )</td>
</tr>
<tr>
<td>( \frac{3}{4} \times \frac{3}{5} = )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \frac{2}{4} \times \frac{5}{6} = )</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(continued on next page.)
3 Solve each of the multiplication problems below. For each:
• outline a rectangle on the grid that will work for both fractions.
• draw and label the dimensions and area, and write the answer.
• write the problem and answer in words.

\[
\frac{2}{3} \times \frac{4}{8} = \frac{8}{24} = \frac{1}{3}
\]

Two-thirds of four-eighths is eight twenty-fourths.

\[\frac{1}{3} \times \frac{3}{4} = \]

\[\frac{2}{4} \times \frac{7}{8} = \]

\[\frac{2}{3} \times \frac{8}{10} = \]

4 Sara says that to multiply two fractions, all you have to do is multiply one numerator by the other, and multiply one denominator by the other. Do you agree? Why or why not?
Set A9 ★ Independent Worksheet 1

INDEPENDENT WORKSHEET

Picturing Fraction Multiplication

1 Each of the pictures below shows the results of multiplying one fraction by another. Label each of the shaded regions with its dimensions and area. Then write a multiplication equation to match.

\[
\frac{2}{3} \times \frac{3}{4} = \frac{6}{12} = \frac{1}{2}
\]

2 Pedro’s dog, Oso, got into the kitchen last night. Oso saw three-fourths of a meat loaf still in the pan. He ate half of the meat loaf that was there before Pedro stopped him. What part of the meat loaf was still left? Use numbers, words, and/or pictures to solve the problem. Show your work.

Answer: __________ of the meat loaf was still left.
Set A9 ★ Activity 5

Target 1 Fractions

Overview
In this session, students share strategies for multiplying whole numbers by a non-unit fraction in preparation for playing Target 1: Fractions. They learn the game by playing a round against the teacher, and then, practice the game with a partner.

Skills & Concepts
★ Add fractions with unlike denominators, including mixed numbers (5.NF.1)
★ Subtract fractions with unlike denominators, including mixed numbers (5.NF.1)
★ Multiply a whole number by a fraction (5.NF.4a)
★ Interpret multiplication as scaling (resizing) (5.NF.5a and 5.NF.5b)

You’ll Need
★ Target 1: Fraction Record Sheet (page A9.34 run a double-sided class set)
★ 8 Digit cards (page A9.35 run a half-class set on cardstock.)
★ chart paper
★ markers
★ scissors, class set

Advance Preparation Cut apart one set of the Digit cards before the session begins to use when teaching the game.

Note When you represent the symbolic form for a fraction, please use a horizontal bar.

Instructions for Target 1
1. Write the problem $3 \times \frac{4}{5}$ where everyone can see and give students a moment to consider how they might solve for it. Ask students what might make this problem challenging (it contains a non-unit fraction). Ask students to turn and talk with a partner about how they might solve $3 \times \frac{4}{5}$. Then, invite several students to share their thinking. Since the numerator is 4, some students may employ the double-double strategy for multiplication. Since the denominator is 5, some students may convert the fraction to a decimal, in the context of money.

Listen for the following strategies:
• Finding the whole number times a unit fraction and scaling up: $3 \times \frac{1}{5} = \frac{3}{5}; \frac{3}{5} \times 4 = \frac{12}{5}$.
• Finding $\frac{1}{5}$ of 3, doubling that, and then doubling that again to get $\frac{4}{5}$ of 3. $\frac{1}{5}$ of 3 = $\frac{3}{5}$ or 0.60, $\frac{3}{5}$ doubled is $\frac{6}{5}$ and 0.60 doubled is 1.20, $\frac{6}{5}$ doubled is $\frac{12}{5}$ or 2 $\frac{2}{5}$, and 1.20 doubled is 2.40.
• Thinking about $\frac{4}{5}$ as money or a decimal: $\frac{4}{5}$ of a dollar equals $0.80. $0.80 \times 3 = 2.40$.
• Finding $\frac{4}{5}$ of 1, three times: $\frac{4}{5} + \frac{4}{5} + \frac{4}{5} = \frac{12}{5}$ or 2 $\frac{2}{5}$.

Invite students to share these strategies. If no one mentions the strategies listed above, you might consider bringing them up yourself. As students share, represent their strategies on a poster for everyone to see.
2. Introduce the game Target 1: Fractions. Display the Target 1: Fractions Record Sheet Teacher Master where everyone can see.

Briefly summarize the game before playing against the class.
- Each player gets 5 Digit Cards.
- Players choose 3 cards to form a whole number and a fraction that they multiply to get a product as close to 1 as possible.
- Players write their numbers on the record sheet and show how they multiplied them. Their score is the difference between their product and 1.
- The player with the lowest score after five rounds wins the game.
3. Play a game of Target 1 Fractions against the class.

   **Teacher** I’m going to play against you. Alex, will you come up here and be the dealer and represent the class? They will help you as you play.

   **Alex** Sure.

   **Teacher** Give us each 5 Digit Cards. Then, you can go first.

   **Alex** Okay, I have a 2, two 3s, a 5, and an 8. What should I use?

   **Juan** Use the smaller numbers. Then you will get closer to 1.

   **Theran** No, use the bigger numbers because fractions with bigger numbers are smaller.

   **Teacher** Why doesn’t everyone try writing a few problems? You don’t have to solve them, but try to estimate so you have a sense of which ones are bigger and which are smaller. Remember to put one digit as the whole number and two digits as the fraction.

   **Alex** OK, I have one. I’m not sure it is the closest to 1, but it is pretty close. I made a 2 the whole number and $\frac{3}{5}$ the fraction.

   **Teacher** Okay. Write it down on the Record Sheet. How did you find the product?

   **Alex** Well, since it is times 2, I could just double $\frac{3}{5}$. That’s $\frac{6}{5}$ which is also $1 \frac{1}{5}$ which is pretty close to 1. It is just $\frac{1}{5}$ away from 1.

   **Teacher** Great. Write your product and your score for this round on the Record Sheet.

4. Pair students up and ask them to cut apart their Digit Cards and play a round of Target 1 Fractions.

   While students are playing, circulate and pose questions like the following to promote flexible thinking and strategy development while you play:
   - Can you have an improper fraction as your fraction? What happens when you have an improper fraction?
   - What combinations of numbers multiply to exactly 1?
   - What happens when the denominator is a relatively small number?
   - What happens when the denominator is relatively large number?
   - Do any arrangements of the same 3 numbers result in the same product?

5. To close the session, bring the class back together and take a few moments to consider the idea that multiplication is like scaling. Ask student questions like the following to guide your discussion.
   - Were you able to estimate and then compare the size of the products without having to do the actual computation? Can you give an example?
   - What happens to the product when the fraction is greater than one? (the product increases)
   - What happens to the product when the fraction is less than one? (the product decreases).

---

**INDEPENDENT WORKSHEET**

Target 1: Fraction  Record Sheet

PLAYER 1____________________________________  PLAYER 2____________________________________

Game 1

<table>
<thead>
<tr>
<th>Equation</th>
<th>Product</th>
<th>My score</th>
<th>Partner’s Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>ex $\frac{5}{8} \times \frac{2}{8} =$</td>
<td>$\frac{10}{8}$ or $1\frac{2}{8}$</td>
<td>$\frac{1}{4}$</td>
<td></td>
</tr>
<tr>
<td>1 _____ $\times$ ___ =</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2 _____ $\times$ ___ =</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3 _____ $\times$ ___ =</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4 _____ $\times$ ___ =</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5 _____ $\times$ ___ =</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Final Score

Game 2

<table>
<thead>
<tr>
<th>Equation</th>
<th>Product</th>
<th>My score</th>
<th>Partner’s Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>6 _____ $\times$ ___ =</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7 _____ $\times$ ___ =</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8 _____ $\times$ ___ =</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9 _____ $\times$ ___ =</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10 _____ $\times$ ___ =</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Final Score
8 Digit Cards
## More Fraction Multiplication

1. Fill in the chart to solve each of the problems below.

<table>
<thead>
<tr>
<th>Multiplication Equation</th>
<th>Word to Match</th>
<th>Labeled Sketch</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{2}{3} \times \frac{2}{3} = \frac{4}{9}$</td>
<td>two-thirds of two-thirds</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\frac{2}{3} \times \frac{6}{7} =$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\frac{1}{2} \times \frac{4}{6} =$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\frac{3}{4} \times \frac{4}{8} =$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2. Solve each problem.

$\frac{3}{4} \times \frac{2}{4} =$ \hspace{1cm} $\frac{1}{4} \times \frac{3}{6} =$ \hspace{1cm} $\frac{5}{6} \times \frac{1}{2} =$ \hspace{1cm} $\frac{6}{7} \times \frac{3}{5} =$

$\frac{2}{3} \times \frac{4}{5} =$ \hspace{1cm} $\frac{6}{8} \times \frac{1}{2} =$ \hspace{1cm} $\frac{3}{4} \times \frac{1}{3} =$ \hspace{1cm} $\frac{2}{7} \times \frac{2}{4} =$
ACTIVITY

Multiplying Domino Fractions

Overview

In this session, students play three rounds of Domino Fractions to develop fluency with multiplication of fractions. Students begin to estimate the product based on the scale of the factors, working first with proper fractions and then with improper fractions. If there is disagreement between partners, students sketch an area model on journal paper to prove their computation. The player with the larger product in the round wins all four domino cards, but at the end of the game, the more/less die is rolled to determine the winner.

Skills & Concepts

★ Multiply a whole number by a fraction and fraction by a fraction (5.NF.4a)

★ Find the area of a rectangle with fractional side lengths (5.NF.4b)

★ Interpret multiplication as scaling (resizing) (5.NF.5a and 5.NF.5b)

You’ll Need

★ Domino Cards (pages A9.41–A9.43 run a half-class set on cardstock, plus one for display)

★ Student Math Journals or Journal Page Grid (A9.44 optional, run as needed.)

★ more/less die, half a class set plus one for display

★ scissors, class set

Advance Preparation  Cut apart one set of Domino Cards before the session to use during the game.

Note  When you represent the symbolic form for a fraction, please use a horizontal bar. Save the Domino cards for use in Supplement Set A11, Activity 7.

Instructions for Multiplying Domino Fractions

Game 1: Introduction

1. Introduce the game Multiplying Domino Fractions by briefly summarizing the game before playing against the class.

   Teacher  In the first round of this game, we’ll each draw two domino cards, read them as proper fractions, and then multiply our two fractions to determine the product. Before we multiply, though, we’ll make a quick estimate of the product. If we disagree, we’ll sketch an area model to show our thinking. I’ll go first to show you.
Teacher  Hmm, I think my product is going to be less than 1 for sure. \( \frac{2}{3} \) is less than one whole and so is \( \frac{4}{5} \). I'll draw an area model to show you how I am thinking about it.

In solving the problem \( \frac{2}{3} \times \frac{4}{5} \), students use the area model to visualize it as a \( 2 \times 4 \) array of small rectangles each of which has side lengths \( \frac{1}{3} \) and \( \frac{1}{5} \). They reason that \( \frac{1}{3} \times \frac{1}{5} = \frac{1}{3 \times 5} \) by counting squares in the entire rectangle, so the area of the shaded area is \( (2 \times 4) \times (3 \times 5) = \frac{2 \times 4}{3 \times 5} \). Using unit fractions, students can scale the area to include \( \frac{2}{3} \) and \( \frac{4}{5} \) or... by counting squares. Encourage students to explain that the product is less than \( \frac{4}{5} \).

The area model and the line segments show that the area is the same quantity as the product of the side lengths.

2. Invite a student to draw two domino cards and show them to the rest of the class. Give students a moment to estimate the product and then have them turn and talk to a partner about their estimate. Invite one or two students to share their thinking.

3. Then, have students compute the total at their desks and ask the player who drew the cards to give and explain the answer. If necessary, sketch the problem with student input.

4. Write both your product and the class's where everyone can see. Ask which product is greater. The player with the greater product wins all four domino cards.

5. Continue the game until all the cards are gone from the deck. Then, count your cards while the class does the same. The player with the fewest cards roll the more/less die to see who wins the round.

Game 2: Partners
6. Pass out the Domino Card Blacklines and ask students to work together to quickly cut apart their domino cards.

7. Then, have students play a round with their partner. Remind students to estimate the product before they compute the total. Will the total be more or less than one whole? Why? Tell students that if there is a disagreement, the player must sketch an area model to show the product in their student journal.

8. When the cards from the deck are all gone, have the players count their cards and roll the more/less die to see who wins the game. When several groups have finished, have the rest of the groups roll to see who won the game.
Game 3: Improper Fractions

9. Introduce a variation of the game. Tell students that this new game is played much the same way, but this time one domino will be used to create a proper fraction while the second domino is used to create an improper fraction. The fractions will then be multiplied using an area model on grid paper.

There is a bit of strategy at play in the second version of the game, and students who can estimate the size of the products without having to perform the computation will have an advantage for capturing the four dominoes in play. Of course, in the end, the more/less die will decide who actually wins the game!

10. Begin the game by drawing two domino cards. Show them to the class, and think aloud as you choose two fractions and estimate their product.

   **Teacher** I drew 2/5 and 7/4. Or I suppose I could think of it as 5/2 and 4/7, couldn't I? Hmm... I think I'm going to solve 5/2 \(\times\) 4/7. I know that 5/2 is 2 1/2 because 4/2 would be 2 and then there is 1/2 left. And then 4/7 is just over 1/2 because 4/8 would be exactly 1/2. For my estimate, I'm going to think of this as half of 2 1/2, which is 1 1/4.

11. Solve the problem and make an area model sketch to show the product.

12. Invite a student to draw two dominoes and create a proper and an improper fraction for the class to solve. Tell students to record an estimate in their journals and turn and talk to a partner about their reasoning before they sketch the area model and solve the problem. Circulate to offer support as students work, and have students compare their work with a partner when they finish.

13. When most students have finished, invite one or two students to share their thinking with the class.

14. If students need more support, play another round and model your thinking again before having them solve another problem. If students understand the work, dismiss them to play with a partner.

15. Tell students that, as before, when all the cards from the deck are gone, players will count their cards and roll the more/less die to see who wins the game.
16. At the end of the session, have students turn and talk to a partner about one thing that was particularly challenging for them today or that they learned, and then call on several students to share their reflections.

**Note** You may want to save the domino cards and use them for additional Work Place practice. Save the cards for Supplement Set A11, Activity 7.

**INDEPENDENT WORKSHEET**

See Supplement Set A9 Independent Worksheet 5 on pages A9.57 and A9.58 for more practice with multiplying fractions.
Domino Cards page 2 of 3

[Diagram of domino cards]
Fraction Stories

1 Jake is making cookies. The recipe says he needs three-fourths of a cup of butter, but Jake wants to cut the recipe in half. What is one-half of three-fourths of a cup of butter? Use numbers, words, and/or pictures to solve the problem. Show your work.

2 Mrs. Smith had $\frac{4}{6}$ of a carton of eggs in her refrigerator. She dropped the carton by accident and a fourth of the eggs in the carton broke. How much of a carton of eggs did she have left after she cleaned up the mess? How many eggs was that? Use numbers, words, and/or pictures to solve the problem. Show your work.

3 Write your own story problem to go with this expression. Then solve it. Use numbers, words, and/or pictures to solve the problem. Show your work.

$$\frac{1}{2} \times \frac{2}{3} =$$

4 Rosa bought a bag of apples. After she baked pies, she had $\frac{2}{3}$ of a bag left. Then she gave her cousin $\frac{3}{4}$ of these, which was 9 apples. How many apples did Rosa have to start?
ACTIVITY

Area Word Problems with Mixed Numbers

Overview
During this session, students will solve several word problems designed to help them develop efficient strategies for multiplying fractions by a whole number using an area model. Their work will include situations involving improper fractions and mixed numbers. The story problem context, along with the use of visual models, will help students make sense of the magnitude of the product.

Skills & Concepts
★ Add fractions with unlike denominators, including mixed numbers (5.NF.1)
★ Multiply a whole number by a fraction (5.NF.4)
★ Interpret multiplication as scaling (resizing) (5.NF.5)
★ Solve word problems involving multiplying fractions and mixed numbers using visual fraction models and equations (5.NF.6)

You'll Need
★ Area Word Problems with Mixed Numbers (page A9.47 run 1 copy for display.)
★ Student Journals or Journal Page Grid (page A9.48 optional, run as needed.)

Note When you represent the symbolic form for a fraction, please use a horizontal bar.

Instructions for Area Word Problems with Mixed Numbers
1. Open the session by telling students that today their work multiplying fractions will extend to mixed numbers and improper fractions. Display the Area Word Problems with Mixed Numbers page so only the first word problem is showing and read it together.

2. Invite students to make a sketch to show how they might find the area of this rectangle. After a few minutes, ask them to share their model with a partner. Finally, choose a student who correctly sketched and labeled an area model to share.
Josie  I shaded \( \frac{1}{2} \) of four boxes in the grid. Next I added \( \frac{1}{2} \) four times for a sum of 2. The area of the rectangle is \( 2 \) \( m^2 \). \( \frac{1}{2} \times 4 = \frac{4}{2} = 2 \)

3. Then, show the rest of the Area Word Problems page and review the directions.

4. Depending on the strengths and needs of your class, you may want to have students do one or two more problems with you, and then work in pairs. Allow students who are ready to work independently to do so while you work with those who need additional support.

5. As students work, watch for strategies to share like scaling up from a unit fraction or using doubling, repeated addition, or money or decimals. Are students able to estimate and then compare the size of the products without having to do the actual computation? Can they explain what happens to the product when the fraction is greater than 1 or what happens when the fraction is less than 1? (the product increases or decreases respectively)

The problems on the Area Word Problems with Mixed Numbers page lend themselves to the following strategies:

- doubling,
- repeated addition
- money/decimals
- scaling up from unit fraction

6. With about 15 minutes left, call the class back together to discuss several of the problems. Choose one of the problems most students finished and invite several students to explain their thinking. If you have time, repeat with another problem, trying to showcase several strategies for each.

7. As you examine strategies for multiplying fractions, consider comparing and contrasting the procedures for adding, subtracting, and multiplying fractions. Why does the algorithm work when we multiply the numerators and denominators across, when that algorithm doesn't work with addition or subtraction of fractions.
Area Word Problems With Mixed Numbers

Here are five problems for you to solve. For each one,
• write the problem number in your journal
• record an estimate (nearest whole number) and explain how you got it
• make a labeled sketch to show your thinking
• write a multiplication equation to match, including the answer

1 A rectangle is 4 meters long and $\frac{1}{2}$ meter wide. What is the area?

2 A painting in the county fair measures 2 meters by $4 \frac{1}{4}$ meters. What is the area of the painting?

3 A teacher measured his classroom door and found that it was $1 \frac{1}{3}$ meters wide and 3 meters tall. What's the area of the door?

4 The rectangular top of a table is three times as long as it is wide. Its width is $1 \frac{2}{3}$ meters. Find the area of the table-top.

5 A small city park consists of a rectangular lawn that is $30 \frac{1}{7}$ long and 20 meters wide. What is the area of the lawn?

6 Kale built a backyard pen for his new puppy. The length of the pen is $6 \frac{1}{4}$ meters and the width is 4 meters. What is the area of the pen?
Using Strategies to Multiply Fractions with Mixed Numbers

Use one of the strategies you know to multiply these problems:

Example $3 \times \frac{4}{5}$

- Finding $\frac{4}{5}$ of 1, 3 times: $\frac{4}{5} + \frac{4}{5} + \frac{4}{5} = \frac{12}{5}$ or $2 \frac{2}{5}$.
- Finding the whole number times a unit fraction and scaling up: $3 \times \frac{1}{5} = \frac{3}{5}$; \[ \frac{3}{5} \times 4 = \frac{12}{5} \]
- Thinking about $\frac{4}{5}$ as money or a decimal: $\frac{4}{5}$ of a dollar equals $0.80$. $0.80 \times 3 = 2.40$

1 Nate is playing Target 1: Fractions. He is trying to solve the following problem:

a $7 \times \frac{2}{3}$. Solve Nate's problem and show your work.

b Nate's partner, Irie, solved $\frac{3}{4} \times 5$. Show how you would solve it.
Independent Worksheet 4  Using Strategies to Multiply Fractions with Mixed Numbers (cont.)

C Irie could have solved $3 \times \frac{4}{5}$ instead of $\frac{3}{4} \times 5$. Which problem is closer to 1?

d Who will win this round? How do you know?
Set A12 ★ Activity 2

Reviewing the Sharing & Grouping Interpretations of Division

Overview
During this activity, students and teacher review two different interpretations of division—grouping and sharing. The computation and the story problems throughout the activity are deliberately simple, involving only whole numbers, to allow students to investigate and understand both interpretations of division.

Skills & Concepts
★ Find whole-number quotients of whole numbers using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. (CCSS 5.NBT.6)
★ Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. (CCSS 5.NBT.6)

You’ll need
★ Poster Problems (page A12.12, run 1 copy on paper, see Advance Preparation)
★ Two Interpretations of 15 ÷ 3 (page A12.13, run 1 copy for display)
★ Grouping or Sharing? (pages A12.14 and A12.15, run 1 copy for display and a class set on paper)
★ Students’ math journals or 1 piece of scratch paper per student
★ colored tile (15 per student)
★ 15 tile for display
★ 2 sheets chart paper (see Advance Preparation)
★ marking pens

Advance Preparation Run a copy of the Poster Problems Blackline. Trim the problems and glue one to the top of each piece of unlined chart paper. Display the charts where all the students can see them.

Instructions for Reviewing the Sharing & Grouping Interpretations of Division
1. Open today’s instruction by letting students know that you’re going to take a day to investigate two different ways to think about division before starting to work on problems that involve dividing with fractions.

2. Have helpers place a container of tile at each table or cluster of desks as you write the expression 15 ÷ 3 on the board. Ask students to each model the expression with tile. After they’ve had a minute or two to work, have them share their work with the people sitting next to them. Then call on a volunteer to build his model at the display and explain his thinking to the class.
Craig  I got 15 tiles and divided them into 3 groups because it says to divide by 3. There were 5 in each group when I got finished.

3. Ask the students if anyone got a different answer or had a different way of thinking about the expression. If no one responds, explain that there are two different ways to think about division. The first is called a sharing interpretation because the total number of objects is shared out evenly among a certain number of groups, 3 in this case. The second is called a grouping interpretation. Model this interpretation yourself if it hasn't come from any of the students by counting out 15 tile as the students watch and then pulling off groups of 3 until none of the tile remain in the original group.

4. Discuss the grouping interpretation with the students. In what way does this interpretation show $15 \div 3$? Is the answer still 5? If so, 5 what?

Alicia  I get the way Craig showed us. With that way, you just take 15 and split it into 3 groups. You get 5 in each group. That’s the answer. I don’t get that other way you showed.

Xavier  I think I get it. With the other way, you start with 15, right? Then you see how many groups of 3 you can make. It’s 5. You can make 5 groups of 3. The answer is still 5, but it means how many groups instead of how many tile in each group.

5. Follow the discussion by placing the Two Interpretations of $15 \div 3$ master. Give students a few moments to examine the master quietly. Then work with the students to determine and record the meaning of each number in the equation for each model.

Teacher  Let’s take a look at the sharing interpretation of division first. In this interpretation, what does the number 15 mean?

Students  It’s the number you start with.
15 tells the number of tile you have to start with.

Teacher  And what does the number 3 mean in this interpretation of division? Talk to the person next to you about this, and show thumbs up when you know. Cory and Mariah, what did you decide?

Mariah  We said the 3 tells you how many groups to split the tiles into.

Cory  It’s kind of like it tells you how many people are going to share 15 tiles or something.

Teacher  And what about the 5? What does that mean in the sharing interpretation?
**Students**  That's the answer. 
*It tells how many each group gets. 
It's like if you had 15 of something and shared it with 3 kids, each kid would get 5.*

Here's how the master might look when you and the class have completed step 5.

![Diagram of Two Interpretations of 15 ÷ 3]

<table>
<thead>
<tr>
<th>Sharing</th>
<th>Grouping</th>
</tr>
</thead>
<tbody>
<tr>
<td>15 means how many you have to start with.</td>
<td>15 means how many you have to start with.</td>
</tr>
<tr>
<td>What does each number mean?</td>
<td>What does each number mean?</td>
</tr>
<tr>
<td>15 ÷ 3 = 5</td>
<td>15 ÷ 3 = 5</td>
</tr>
<tr>
<td>3 tells you how many groups to split the 15 into. (It's like telling you how many people to share with.)</td>
<td>3 tells you how many to put in each group.</td>
</tr>
<tr>
<td>5 is the answer. It tells how many tile each group gets.</td>
<td>5 is the answer. It tells how many tiles each group gets.</td>
</tr>
</tbody>
</table>

6. Then draw students’ attention to the two charts you posted. Read the story problem at the top of each sheet with the class, and solicit the answer to both—3. Pose the following question before you move along to the next step: Even though the answer to both problems is 3, does the 3 mean the same thing in each case?

7. Rather than taking time to discuss the question with the class right away, ask students to get out their math journals, or give them each a piece of scratch paper to work on. Have them:
   - Draw a line down the middle of the next available journal page or sheet of paper, and label the top of one side *sharing* and the other side *grouping*.
   - Have them model each situation using simple dots and loops.
   - Ask them to label each visual model with an equation to match, and explain, in writing, the meaning of each number in the equation.

8. When students have had 7–10 minutes to work on the task, reconvene the class. Ask one volunteer to describe her model for the first problem. Then have the other students explain the meaning of each number in the matching equation. Repeat this with the second problem. As students are explaining, record their thinking on the two pieces of chart paper.
9. As you discuss the two problems with the class, pose the following questions and prompts, and record students’ responses on the charts:
- What information is provided in each problem, and what do you have to find out?
- What is the key question in each problem?
- What interpretation of division does each story problem illustrate? (If your students want to devise their own names for the sharing and grouping strategies, that’s fine.)

Jeff made 12 cookies, and divided them equally among his 4 cousins. How many cookies did each cousin get?

Jeff made 12 cookies and packed them into sacks of 4. How many sacks of cookies was he able to make?

Kid 1 Kid 2 Kid 3 Kid 4

Key Question:
How many cookies does each kid get? (SIZE of groups)

10. Give students each a copy of Grouping or Sharing? and display the masters for everyone to see. Review and discuss the tasks with the class. Give students the option of working on these sheets independently, in pairs, or with you.

11. As students finish, have them share and compare their work with a classmate. If there are differences in some of their responses, have them work together to resolve those differences or check with a third classmate before consulting you. Have them check each others’ story problems to be sure each correctly illustrates the designated interpretation of division.

If students need more time to complete the assignment, ask them to finish it as homework or as part of their morning seatwork the following day before you conduct Activity 3.

Note Leave the story problem posters you made with the class today on display for students’ reference throughout Activities 3–7 in this supplement set.
Activity 2  Reviewing the Sharing & Grouping Interpretations of Division (cont.)

INDEPENDENT WORKSHEET

See Set A12 Independent Worksheet 1 on page A12.75 for more practice with identifying sharing and grouping situations, identifying the operation needed to solve a story problem, and multiplying fractions and whole numbers.
### Poster Problems

<table>
<thead>
<tr>
<th>Jeff made 12 cookies, and divided them equally among his 4 cousins. How many cookies did each cousin get?</th>
<th>Jeff made 12 cookies and packed them into sacks of 4. How many sacks of cookies was he able to make?</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>3</td>
</tr>
</tbody>
</table>
Two Interpretations of $15 \div 3$

Sharing

What does each number mean?

$15 \div 3 = 5$

Grouping

What does each number mean?

$15 \div 3 = 5$
1 Decide whether each of the story problems below involves the sharing or the grouping interpretation of division. Circle your choice. You don't need to solve the problems, but it may help to think about whether the answer will mean how many items are in each group or how many groups can be made.

a Frank picked 12 flowers. He divided the flowers evenly between 3 vases. How many flowers did he put in each vase?

Sharing                    Grouping

b Erica had 20 baseball cards. She put them up on her bulletin board in rows containing 5 cards. How many rows was she able to make?

Sharing                    Grouping

c Darius and his dad made 28 cupcakes for the bake sale. They put 7 cupcakes on each plate. How many plates of cupcakes were they able to make?

Sharing                    Grouping

d Kiara and her sister are collecting pennies. They have 120 pennies so far. They put their pennies into stacks of 10. How many stacks were they able to make?

Sharing                    Grouping

e Carlos has 15 matchbox cars. He wants to share them with 2 of his friends so all three boys have the same number. How many cars will each boy get?

Sharing                    Grouping

f Jade and her sister made $5.00 doing chores for their mom. They split the money evenly between themselves. How much money did each girl get?

Sharing                    Grouping

(Continued on next page.)
Grouping or Sharing? Page 2 of 2

2 Read each story problem. Then:
• write an equation (including the answer) for the problem.
• fill in the bubble to show whether the answer means the size of each group or the number of groups

a Mai and her mother made 24 invitations to Mai’s birthday party. They put the invitations into stacks of 4. How many stacks did they make?

Equation: ________________________________

The answer means:
○ the size of each group (for example, the number of items each person got)
○ the number of groups

b Troy and his mom got a case of bottled water for the soccer game. They divided the 24 bottles evenly among the 12 boys on the team. How many bottles of water did each boy get?

Equation: ________________________________

The answer means:
○ the size of each group (for example, the number of items each person got)
○ the number of groups

3 Choose one of the expressions below and circle your choice. Write a sharing story problem and a grouping story problem about the same expression.

<table>
<thead>
<tr>
<th>Sharing Story Problem</th>
<th>Grouping Story Problem</th>
</tr>
</thead>
<tbody>
<tr>
<td>18 ÷ 6</td>
<td>60 ÷ 12</td>
</tr>
<tr>
<td>108 ÷ 4</td>
<td>400 ÷ 25</td>
</tr>
</tbody>
</table>
Set A9 ★ Independent Worksheet 5

INDEPENDENT WORKSHEET

Domino Multiplication

1 Write the two fractions below the dominoes and then multiply them to find the product. Show your work, and reduce the fraction if you can.

- \[
\begin{array}{c}
\text{a} \\
\hline
\frac{\text{ } }{\text{ } } \times \frac{\text{ } }{\text{ } } = \frac{\text{ } }{\text{ } }
\end{array}
\]

- \[
\begin{array}{c}
\text{b} \\
\hline
\frac{\text{ } }{\text{ } } \times \frac{\text{ } }{\text{ } } = \frac{\text{ } }{\text{ } }
\end{array}
\]

- \[
\begin{array}{c}
\text{c} \\
\hline
\frac{\text{ } }{\text{ } } \times \frac{\text{ } }{\text{ } } = \frac{\text{ } }{\text{ } }
\end{array}
\]

- \[
\begin{array}{c}
\text{d} \\
\hline
\frac{\text{ } }{\text{ } } \times \frac{\text{ } }{\text{ } } = \frac{\text{ } }{\text{ } }
\end{array}
\]

(Continued on next page.)
Independent Worksheet 5  Domino Multiplication (cont.)

2 Write a multiplication word problem that matches the following dominoes and solve it.

a

b

CHALLENGE

3 Now invert one of the dominoes in each set to create a new improper fraction and then multiply the two fractions to find the product. Remember to show your work!
Set A12 ★ Activity 3

Grouping Stories

Overview
Today’s activity opens with a short string of division problems. The last problem in the set involves division of a whole number by a fraction. Students review the sharing and grouping interpretations of division, decide which makes more sense in this context, and test their decision by solving and discussing two related story problems. The activity concludes with a story designed to help students make generalizations about the effect of dividing a whole number by a fraction.

You’ll need
★ Division String (page A12.25, run 1 copy for display)
★ Two Problems (page A12.26, run 1 copy for display)
★ Division Posters from Activity 2 (displayed where all the students can see them)
★ Students’ math journals or 1 piece of scratch paper per student
★ class set of whiteboards, pens, and erasers or a second piece of scratch paper per student
★ marking pens
★ a piece of paper to mask portions of the display
★ a clean, empty half-gallon milk carton (optional, see Advance Preparation)
★ 2 one-quart bottles from the Grade 5 Number Corner Kit (optional, see Advance Preparation)
★ 1-cup and 2-cup liquid measuring cups from the Grade 5 Number Corner Kit (optional, see Advance Preparation)
★ small bottle of food coloring (optional, see Advance Preparation)
★ empty clean plastic bowls or containers in the following numbers: 4 two-cup containers, 8 one-cup containers, 16 ½-cup containers (optional, see Advance Preparation)

Skills & Concepts
★ Divide a whole number by a unit fraction using visual models (CCSS 5.NF.7b)
★ Solve story problems involving division of a whole number by a unit fraction (CCSS 5.NF.7c)

Advance Preparation
The half-gallon milk carton, quart bottles, measuring cups, food coloring, and bowls or other containers are optional, and designed to provide support to students who are struggling with the concepts presented in this activity. We suggest you read through the activity, including the support suggestion on the last page, and decide whether or not your students are likely to need a concrete model to help them understand that dividing a whole number by a fraction results in a quotient that’s larger than the dividend. If so, fill the half-gallon milk carton with 8 cups of water and add about 10
Set A12 Number & Operations: Dividing Fractions & Whole Numbers

Activity 3  Grouping Stories (cont.)

drops of food coloring. Arrange the carton of water, the 2 quart bottles and the liquid measuring cups, and the plastic bowls or containers on a tray prior to conducting this activity.

Instructions for Grouping Stories

1. Open today’s instruction by letting students know that you’re going to start with a short review of the two different interpretations of division the class investigated last session—sharing and grouping.

2. Display just the top row of the Division String master, keeping the rest of the sheet covered for now. Read the combination with the class, and have the students supply the answer as you write it in. Then work with input from the class to write a sharing and a grouping story problem to match, using the theme of children and food (or any other theme you choose). Take this opportunity to review the two interpretations of division, using the posters from the previous session for reference. Reflect, also, with the students that the quotients mean two different things—in the first case, the size of each group; in the second, the number of groups.

3. Reveal the next two equations on the master, one by one, and repeat step 2. Then reveal the final expression—6 ÷ 1/2. Read it with the class, and ask students to talk with the people sitting next to them about the answer. After a few moments, invite several volunteers to share their answers with the class. Record all responses without comment, even if one or more of them are incorrect.

4. Since at least some of your students will likely give 3 as an answer, press them to explain their thinking. How can 6 ÷ 2 and 6 ÷ 1/2 yield the same result?

   Teacher  Many of you are saying that the answer to 6 ÷ 1/2 is 3. Would someone be willing to explain why?

   Marcus  Simple—1/2 of 6 is 3.

   Teacher  So, you’re reading the expression 6 divided by 1/2 to mean 1/2 of 6? Thumbs up if you agree with Marcus. Kiara, would you like to comment on this?

   Kiara  I agree with Marcus. I don’t know what else it could mean. I think 6 ÷ 1/2 means that you should cut 6 in 1/2, and that’s 3.

   Teacher  Okay, I’m confused. Earlier, we all decided that six divided by two is three. How can six divided by 1/2 give us the same answer?
5. Resist the temptation to correct student thinking at this point. Instead, move along to posing story problems that represent $6 \div \frac{1}{2}$. Using the first three sharing problems on the master for reference (as well as the division posters from the previous session if they seem helpful), solicit agreement from the students that a sharing problem for $6 \div \frac{1}{2}$ would be like sharing 6 with half a person. Maybe grouping makes more sense for $6 \div \frac{1}{2}$.

6. After a minute or two of discussion and debate, table the attempt to write a sharing problem, and explore the possibility of writing a grouping problem to match the combination. This time, solicit agreement from the students that a grouping problem for $6 \div \frac{1}{2}$ would mean dividing 6 into groups of $\frac{1}{2}$; in other words, determining how many halves there are in 6. Is this possible?

If the students are able to come to some sort of resolution about the question and generate a valid grouping problem for $6 \div \frac{1}{2}$, record it in the correct box on the master. If not, leave the question unresolved for now, and move on to step 7. (While some students may quickly see that $6 \div \frac{1}{2}$ is 12, and come up with a story problem to match, others will likely be quite convinced that $6 \div \frac{1}{2}$ is 3, and not easily dissuaded.)

7. Whether or not the class reached resolution about the meaning of $6 \div \frac{1}{2}$, explain that you have a couple of story problems to pose that might further their thinking on the issues raised. Display the Two Problems master as students get out their math journals or helpers pass out scratch paper.

8. Give students a few moments to examine the sheet quietly. Then read and review the instructions and the problems, providing clarification as needed without telling the students how to solve either of the problems. Be sure to emphasize the requirement that they make a visual fraction model to show and solve the problem they choose.
9. When students understand what to do, have them go to work. Circulate to observe and provide clarification if necessary, but avoid the temptation to tell students how to model or solve the problems. (Encourage students who may be getting frustrated at this point to see if they can draw a picture that might help them understand the situation. These students might also be encouraged to see how others around them are handling the problems.)

10. As they finish one of the problems, ask the students to share and compare their work with a classmate, and then either solve the other problem or work on the challenge associated with the problem they chose first.

11. After about 10 minutes, reconvene the class, even if some of the students haven’t finished. Solicit and record all answers to the first problem. Then invite several volunteers to share their work with the class, either at the display, or by replicating their visual model on the board and explaining it to the class.

Carlos: I just drew a path from the snail’s house to his grandma’s, like this. I put him here, and his grandma 6 miles away. Then I marked off half-miles to see how many days it would take him, like this. After that, I counted up the number of halves—that’s what those checkmarks are for, and it came out to 12.
Teacher  Does anyone have any questions for Carlos? No? I do. Carlos, how many days did you decide it would take the snail to get to his grandma’s?

Carlos  I think it must be 12.

Amber  I agree with Carlos. I drew a picture kind of like his. Then I saw that there were 2 half-miles in every mile, so I just said, okay, if the snail can go one half mile every day, then it would be 2, 4, 6, 8, 10, 12 days for him to get there.

Teacher  Did anyone have a different visual model to represent and solve this problem?

DeAndre  My idea is kind of the same, but I did it one mile at a time, like this.

Teacher  I have another question for those of you who are thinking it took the snail 12 days to get to his grandma’s. What equation did you write to represent the problem?

Carly  I got 12, and I wrote $6 \div \frac{1}{2} = 12$.

Teacher  Can someone repeat Carly’s equation for me? I want to record it on the board.

Sam  She said $6 \div \frac{1}{2} = 12$, but I respectfully disagree with the answer. I said $6 \div \frac{1}{2}$, but I think the answer is 3.

Teacher  Did you make a visual model to show and solve the problem, Sam?

Sam  Yes. First I wrote the equation, and I know the answer, so then I made the picture. See? Half-way to 6 is 3, so it took the snail 2 days.
Teacher  Comments or questions for Sam, anyone?

Jasmine  I get what you did, Sam, but if it only took 2 days, that means the snail had to go 3 miles a day, and the problem says he could only go half a mile a day.

12. Once several students have shared their work, move along to the second problem, even if there wasn't consensus on the answer to the first. Again, solicit and record answers, and invite a few volunteers to share their models and strategies with the class. (Despite the fact that students who are able to model the situations accurately will arrive at an answer of 12, there may still be a few who are confused or convinced that the answer is 3 because 3 is half of 6.)

13. When you have discussed both problems with the class, question them closely about the answer most of them may agree upon at this point. When we divide, we usually get an answer that's smaller than the dividend. How can one possibly start with 6 and get an answer of 12?

Students  Think about it with cookies. If you have 6 cookies and break each one of them in \( \frac{1}{2} \), you get 12 halves, right?
Or if you have 6 sandwiches, and you cut each one in \( \frac{1}{2} \), you get 12.
I think if you break something into groups that are smaller than 1, you get more of them.
I don't get it—I just don't.

14. Explain that you are going to finish up today's lesson by telling a story that may give students another way to think about dividing by whole numbers and fractions. (Even if many of your students grasp the situation clearly at this point, their understandings are likely fragile, and it's quite possible that a few are still confused by the idea of dividing a whole number by a fraction.)

15. Give each student a whiteboard, pen, and eraser, or a piece of scratch paper, and explain that as you tell the story, they'll write a set of equations to match. Ask them to start at the top of their board or paper and write large enough for you to see, but leave enough room to record several equations in a list. Then start the story, using or adapting the text below as you see fit.

One day, Anthony decided to make a pot of soup. First, he set a nice batch of beans on the stove to cook. Then he started chopping up onions, carrots, celery, and potatoes to add to the beans. As he was working, he started thinking about inviting some friends over to share the soup with him. By the end of the afternoon, he had 8 cups of bean soup. He was very hungry by then, so he got out his biggest soup bowls, and thought, “This soup is so good, and I’m so hungry. I think I’ll give everyone a 4-cup serving. How many people could Anthony serve”

16. Stop at this point and ask students to record a division equation to match the situation. Have them hold up their boards or papers, and then call on a volunteer who has correctly recorded \( 8 \div 4 = 2 \) to share and explain her equation. As she does so, record it on the board, and review the meaning of each
of the numbers with the class (8 is the number of cups of soup Anthony made. Four is the serving size; he's planning to give each person a 4-cup serving. Two is the number of people he can serve.) Ask her if it's a sharing or grouping problem and why. Then continue the story.

When Anthony realized that he could only make two 4-cup servings, he said, “That means I can only invite 1 other person besides myself. That’s not much of a party. Maybe I’ll give each person a 2-cup serving instead.” How many people could Anthony serve now?

17. Pause again and have students write a second equation under the first to represent the situation. Have a different volunteer share and explain his equation. Record it on the board, and review the meaning of each of the numbers with the class. Then continue the story.

When Anthony realized that he could only make four 2-cup servings, he said, “That means I can only invite 3 other people besides myself. I have more friends than that. Maybe I’ll give each person a 1-cup serving instead.”

18. Pause again and have students write a third equation to represent the situation. Have a different volunteer share and explain her equation. Record it on the board, and review the meaning of each of the numbers with the class. Then discuss the set of equations so far:
• What patterns, if any, can students find in the numbers?
• What is the effect of decreasing the serving size?

\[
\begin{align*}
8 \div 4 &= 2 \text{ people} \\
8 \div 2 &= 4 \text{ people} \\
8 \div 1 &= 8 \text{ people}
\end{align*}
\]

**Students**  He has 8 cups of soup every time.
The serving sizes are getting smaller. First he was going to give everyone 4 cups, then 2, and now just 1 cup of soup.
The numbers you're dividing by are getting cut in half—4, then 2, now 1.
And the answers are doubling! Look—2, 4, and now 8.
When you make smaller servings, you can invite more people.

19. Continue the story:

Anthony was in a dither. He didn't have time to make any more soup, but 7 people besides himself didn't come close to the number of friends he had in the village. He walked around and around in circles trying to figure out what to do. Suddenly he stopped, smacked his forehead and said, “I’ve got it! I’ll give everyone just half a cup of soup. That will mean I can have 15 people and myself—16 servings!”
20. Have students record and discuss the matching equation. Does it work? Is it true that if Anthony reduces the serving size to half a cup each, he can have a total of 16 people (including himself) at his party? Have students share thoughts about the situation, first in pairs and then as a whole group.

\[
\begin{align*}
8 \div 4 &= 2 \text{ people} \\
8 \div 2 &= 4 \text{ people} \\
8 \div 1 &= 8 \text{ people} \\
8 \div \frac{1}{2} &= 16 \text{ people}
\end{align*}
\]

**Students**  
It fits with the pattern, because you keep cutting the amount of soup in half, and you can have twice as many people.  
There's 2 halves in one cup, right? So there must be 16 halves in 8.  
All you have to do is multiply by 2 to get the answer!  
Weird—I don't get it.  
I think I do now. If you go \(8 \div 2\), and think about how many 2s there are in 8, it's 4. If you go \(8 \div \frac{1}{2}\), and think about how many \(\frac{1}{2}\)-cups there are in 8, it's 16.  
You have to use division the grouping way! If you think about the sharing way, it doesn't even make sense. You can't share 8 with half of a person!

**Support**  
If more than a very few students seem confused at this point, do a live demonstration with the half-gallon of colored water, liquid measuring tools, and bowls or plastic containers, as described here:  
- Review the fact that there are 8 cups in a half gallon. Explain that the water in the half-gallon milk carton you've prepared represents Anthony's batch of soup.  
- Go back to the first part of the story and the first equation, in which Anthony decides to give each person a 4-cup serving of soup. Pour 4 cups of colored water from the milk carton into each of two quart bottles. Solicit agreement from the students that each serving is quite large, and only 2 people can be served.  
- Go back to the second part of the story and the second equation, in which he decides to give each person a 2-cup serving of soup. Pour all the water back into the \(\frac{1}{2}\)-gallon milk carton and then measure 2 cups into each of four smaller containers. Confirm with the students that this results in 4 servings.  
- Pour the water back into the milk carton. Repeat the pouring and measuring process twice more, first creating eight 1-cup servings, and finally creating sixteen \(\frac{1}{2}\)-cup servings.

**Challenge**  
If students are making sense of the situation, and understand that 8 cups of soup divided into \(\frac{1}{2}\)-cup servings makes it possible to serve 16 people, continue the story. Anthony is so big-hearted...
### Division String

<table>
<thead>
<tr>
<th>Equation</th>
<th>Sharing Problem</th>
<th>Grouping Problem</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 $6 \div 3 = _____$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2 $6 \div 2 = _____$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3 $6 \div 1 = _____$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4 $6 \div \frac{1}{2} = _____$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Two Problems

Choose *one* of the two story problems below and copy it into your journal (or onto your piece of paper) Then:

- Use numbers, labeled visual models, and/or words to solve the problem and explain your strategy.
- Write an equation to match.

You will have about 10 minutes to work. If you finish one problem, do the other, or solve the challenge version of the problem you started with.

1a **Little Snail** is going to visit his grandma, who lives 6 miles away. He can crawl $\frac{1}{2}$ mile a day. How many days will it take him to get to his grandma’s house?

Equation: _______________________________________________

1b **Challenge** What if Little Snail crawled $\frac{2}{3}$ of a mile a day? Then how many days would it take him to get to his grandma’s house?

Equation: _______________________________________________

2a How many $\frac{1}{2}$ cup servings are there in 6 cups of ice cream?

Equation: _______________________________________________

2b **Challenge** How many $\frac{3}{4}$ cup servings are there in 6 cups of ice cream?

Equation: _______________________________________________
Set A12 ★ Independent Worksheet 1

Sharing & Grouping  Multiplying & Dividing

1  Read each story problem. Then:
•  write an equation (including the answer) for the problem.
•  fill in the bubble to show whether the answer means the size of each group or
  the number of groups

a  The swim team is going to a meet across town. There are 35 swimmers on the
team, and each van can take 5 of them. How many vans will be needed to take
the whole team?

Equation: _______________________________________________

The answer means:
○  the size of each group (for example, the number of items each person got)
○  the number of groups

b  Jacob picked 28 flowers and divided them equally between 2 vases. How
many flowers did he put in each vase?

Equation: _______________________________________________

The answer means:
○  the size of each group (for example, the number of items each person got)
○  the number of groups

2  Circle the equation that matches each story problem. Then fill in the correct
answer.

a  Alexus and her two sisters picked 48 strawberries and shared them equally.
How many strawberries did each girl get?

\[ 48 \div 2 = \underline{\phantom{0}} \quad 3 \times 48 = \underline{\phantom{0}} \quad 48 \div 3 = \underline{\phantom{0}} \quad 48 - 3 = \underline{\phantom{0}} \]

(Continued on next page.)
b  Miguel is making valentines. It takes $\frac{1}{2}$ of a sheet of paper for each valentine, and Miguel wants to make 26 valentines. How many sheets of paper will he need?

$$26 \div \frac{1}{2} = \_\_\_ \quad 26 \times \frac{1}{2} = \_\_\_ \quad 26 \times 2 = \_\_\_ \quad 26 - \frac{1}{2} = \_\_\_$$

c  Ling and her mother are making dumplings. It takes $\frac{3}{4}$ of an ounce of meat for each dumpling, and they are going to make 36 dumplings. How many ounces of meat will they need?

$$36 \times 4 = \_\_\_ \quad 3 \times 36 = \_\_\_ \quad \frac{3}{4} \div 36 = \_\_\_ \quad 36 \times \frac{3}{4} = \_\_\_$$

d  There was $\frac{1}{2}$ of a pan of cornbread leftover from dinner. Jake and his dad ate half of the leftover cornbread for breakfast. How much of the whole pan did they have at breakfast?

$$\frac{1}{2} \times \frac{1}{2} = \_\_\_ \quad \frac{1}{4} + \frac{1}{4} = \_\_\_ \quad \frac{1}{2} \div \frac{1}{2} = \_\_\_ \quad \frac{1}{2} - \frac{1}{2} = \_\_\_$$

3  Each of the visual models below shows the results of multiplying one fraction by another. Label each of the shaded regions with its dimensions and area. Then write a multiplication equation to match.

**ex**

Equation $\frac{2}{3} \times \frac{3}{4} = \frac{6}{12} = \frac{1}{2}$

**a**

Equation

**b**

Equation

**c**

Equation
Set A12 ★ Activity 4

ACTIVITY

Dividing a Whole Number by a Fraction

Overview
After a short review of the sharing and grouping models of division, students work independently to solve two story problems. The teacher reconvenes the class and works with the students to record several strategies for dividing whole numbers by unit fractions. Students then complete a related assignment.

Skills & Concepts
★ Divide a whole number by a unit fraction using visual models (CCSS 5.NF.7b)
★ Create a story context for dividing a whole number by a unit fraction (CCSS 5.NF.7b)
★ Solve story problems involving division of a whole number by a unit fraction (CCSS 5.NF.7c)

You’ll need
★ Two More Problems (page A12.33, run 1 copy for display)
★ Do-It-Yourself Story Problems (pages A.12.34–36, run 1 copy for display and a class set on paper)
★ Division Posters from Activity 2 (displayed where all the students can see them)
★ Students’ math journals or 1 piece of scratch paper per student
★ 2 pieces of chart paper
★ marking pens

Instructions for Dividing a Whole Number by a Fraction
1. Open today’s instruction by letting students know that you’re going to start with a short review of the two different interpretations of division the class investigated last session and the one previous—sharing and grouping.

2. Draw students’ attention to the two posters created by the class during Activity 2. Give students a few moments to study the posters quietly. Then write 10 ÷ 2 on the board and ask them to brainstorm, first in pairs and then as a whole group, a sharing and a grouping story problem to match. Record an example of both types of story problems on the board, along with the solution to each. Note with students that the quotient means two different things: the size of the group in the case of the sharing problem, and the number of groups in the grouping problem.

10 ÷ 2

Sharing Story Problem
I had 10 marbles. I split them evenly with a friend. How many marbles did we each get?
Answer: 5 marble each

Grouping Story Problem
I had 10 socks. I put them into pairs. How many pairs was I able to make?
Answer: 5 pairs of socks
3. Next, write $10 \div \frac{1}{2}$ on the board and read it with the class. Then go back and read it to the class as a sharing and then a grouping situation. Which interpretation makes better sense?

   **Teacher**  Okay, so we agree that this expression says ten divided by $\frac{1}{2}$. I’m going to phrase it in two different ways now—sharing first, and then grouping. Think privately about which version makes more sense, and then we’ll talk.

   Ten shared evenly with half a person.
   Ten split into groups of a half.

   **Students**  You can’t share something with half a person.
   I think you have to use the grouping way.
   It’s 5! Just split 10 in half, and it’s 5!
   But this is different. If you think about grouping, it’s like asking how many groups of a half are in 10.
   Right! Half of 10 is 5!
   That’s right, but I’m talking about halves. Little halves, smaller than 1. There are two halves in 1 thing, right? So this is like how many halves in 10 things?

4. Even if a few students are still somewhat confused by the situation, confirm what many probably understand by now—when you divide a whole number by a unit fraction, the grouping interpretation better sense than the sharing interpretation. $10 \div \frac{1}{2}$ requires the problem-solver to figure out how many halves, as in a half of 1, are in 10.

5. Have students brainstorm, first in pairs and then as a whole group, a grouping story problem to match $10 \div \frac{1}{2}$. Record one of their ideas on the board, and work with the class to solve the problem.

6. Next, display the Two More Problems master as students get out their math journals or scratch paper.

7. Give students a few moments to examine the sheet quietly. Then read and review the instructions and the problems, providing clarification as needed without telling the students how to solve either of the problems. Be sure to emphasize the requirement that they make a labeled model to show and solve the problem they choose.
Two More Problems

Choose one of the two story problems below and copy it into your journal (or onto your piece of paper). Then:
- create a labeled visual model to represent and solve the problem.
- write an equation to match.

You will have about 10 minutes to work. If you finish one problem, do the other, or solve the challenge version of the problem you started with.

1a. It takes \( \frac{1}{3} \) of a yard of ribbon to make a hair bow. Maya has 5 yards of ribbon. How many hair bows can she make?

Equation: _______________________________________________

1b. Challenge. Fancy hair bows take \( \frac{3}{4} \) of a yard of ribbon each. How many fancy hair bows can Maya make with 5 yards of ribbon?

Equation: _______________________________________________

2a. Our aquarium holds 5 liters of water. If we use a scoop that holds \( \frac{1}{6} \) of a liter of water, how many scoops will be needed to fill the aquarium?

Equation: _______________________________________________

2b. Challenge. If we use a larger scoop that holds \( \frac{2}{3} \) of a liter of water, how many scoops will be needed to fill the aquarium?

Equation: _______________________________________________

Note. The challenge problems are optional, and push beyond the expectation that fifth graders will learn to divide whole numbers by unit fractions, as both involve proper fractions and remainders. You likely have a few students who understand fractions well enough to devise visual strategies that make sense and result in the correct answers. If they get stuck, the attempt is still valuable. Rather than telling them how to solve these problems, encourage them to use visual models in order to see and make sense of the situations involved. See the second note below step 10 for examples of such student-generated strategies.

8. When students understand what to do, have them go to work. Circulate to observe and provide clarification if necessary, but avoid the temptation to tell students how to model or solve the problems. (Encourage students who may be getting frustrated at this point to see if they can draw a picture that might help them understand the situation. These students might also be encouraged to see how others around them are handling the problems.)

9. As they finish one of the problems, ask the students to share and compare their work with a classmate, and then either solve the other problem or work on the challenge associated with the problem they chose first.

10. After about 10 minutes, reconvene the class, even if some of the students haven't finished. Solicit and record all answers to the first problem. Then invite several volunteers to share their work with the class, either at the display, or by replicating their visual model on the board and explaining it to the class. Repeat this process with the second problem.
Activity 4  Dividing a Whole Number by a Fraction (cont.)

**Note**  The illustration below shows a strategy for each of the challenge problems, along with the correct answer. These, again, are representative of the types of strategies capable students may use.*

1b Challenge: Fancy hair bows take $\frac{3}{4}$ of a yard of ribbon each. How many fancy hair bows can Maya make with 5 yards of ribbon?

There’s $\frac{1}{4}$ a yard of ribbon still left over.

$5 \div \frac{3}{4} = 6 \text{ r } \frac{1}{4}$

She can make 6 hair bows, and there’s $\frac{1}{4}$ a yard of ribbon left over.

2b Challenge: If we use a larger scoop that holds $\frac{2}{3}$ of a liter of water, how many scoops will be needed to fill the aquarium?

It’s 7 scoops that are each $\frac{2}{3}$ of a cup. There’s still a $\frac{1}{2}$ of a cup left to fill at the end. Since $\frac{1}{2}$ is half of $\frac{2}{3}$, the answer is $7\frac{1}{3}$ scoops.

After each volunteer has shared, work with input from the class to replicate the model and summarize the strategy used to solve the problem on a piece of chart paper.
Note: The strategies shown on the charts above represent some of the ways fifth graders are likely to deal with dividing whole numbers by fractions in the context of selected story problems. Some of these strategies are more efficient than others, and it’s entirely possible that your students will devise methods that are not shown above. It should be stressed that you are not responsible for teaching any of these strategies directly. The goal of your instruction at this point should be to provide students with opportunities to make sense of situations that involve division of whole numbers by fractions; to ask that they bring perseverance, a willingness to take risks, and the knowledge they already possess about division and fractions to bear on problems that require them to stretch a little (or a lot, in some cases).

11. Give students each of copy of the Do-It-Yourself Story Problems and display the masters for everyone to see. Review and discuss the tasks with the class. Give students the option of working on these sheets independently, in pairs, or with you. Encourage students to make use of the strategies just shared by their classmates.

12. As students finish, have them share and compare their work with a classmate. If they have selected different divisors or dividends for each problem, challenge them to be sure each other’s answers are correct. Have them check each other’s story problems to be sure they make sense and have been solved correctly.

If students need more time to complete the assignment, ask them to finish it as homework or as part of their morning seatwork the following day before you conduct Activity 5.
Note  Save the strategy posters from today's activity for use in Activity 7.

INDEPENDENT WORKSHEET

See Set A12 Independent Worksheet 2 on page A12.77 for more practice with dividing a whole number by a fraction, identifying the operation needed to solve a story problem, and multiplying fractions and whole numbers.
Two More Problems

Choose one of the two story problems below and copy it into your journal (or onto your piece of paper). Then:

• create a labeled visual model to represent and solve the problem.
• write an equation to match.

You will have about 10 minutes to work. If you finish one problem, do the other, or solve the challenge version of the problem you started with.

1a It takes $\frac{1}{3}$ of a yard of ribbon to make a hair bow. Maya has 5 yards of ribbon. How many hair bows can she make?

Equation: _______________________________________________

1b Challenge Fancy hair bows take $\frac{3}{4}$ of a yard of ribbon each. How many fancy hair bows can Maya make with 5 yards of ribbon?

Equation: _______________________________________________

2a Our aquarium holds 5 liters of water. If we use a scoop that holds $\frac{1}{6}$ of a liter of water, how many scoops will be needed to fill the aquarium?

Equation: _______________________________________________

2b Challenge If we use a larger scoop that holds $\frac{2}{3}$ of a liter of water, how many scoops will be needed to fill the aquarium?

Equation: _______________________________________________
Do-It-Yourself Story Problems  Page 1 of 3

1 Solve each of the story problems below. For each problem:
• Choose and circle one of the numbers in parentheses, depending on how challenging you want the problem to be.
• Write an expression to represent your problem.
• Use numbers, labeled visual models, and/or words to solve the problem and explain your strategy.
• Complete the sentence below with your solution to the problem

a It takes ($\frac{1}{2}$, $\frac{1}{4}$, $\frac{3}{4}$, $\frac{3}{8}$) of a cup of sugar to make a batch of cookies. I have 5 cups of sugar. How many batches of cookies can I make?

Expression: _______________________________________________

I can make_______ batches of cookies.

(Continued on next page.)
The road-repair crew can fix \( \frac{1}{2} \) a mile of road per day. How many days will it take them to fix a stretch of road that is (4 miles, 5 miles, 6 \( \frac{1}{2} \) miles, 8 \( \frac{3}{4} \) miles)?

Expression: _______________________________________________

It will take ________ days to fix the road.

How many ( \( \frac{1}{2} \) cup, \( \frac{1}{3} \) cup, \( \frac{1}{4} \) cup, \( \frac{2}{3} \) cup) servings are there in a quart of ice-cream? There are 4 cups in a quart.

Expression ______________________________________________

There are ______ servong in a quart of ice-cream.

(Continued on next page.)
Little Snail is going to visit his friend over at the next pond, 3 miles away. He can crawl (\(\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{3}{4}, \frac{2}{3}\)) of a mile per day. How many days will it take him to get there?

Expression: _______________________________________________

It will take _______ days to get there.

2. Choose one of the expressions below and circle your choice. Write a story problem about the expression you circled. Then solve your own problem using a fractional visual model.

\[10 \div \frac{1}{2} \quad 12 \div \frac{1}{3} \quad 20 \div \frac{1}{4} \quad 24 \div \frac{2}{3}\]

My story problem:

My work:

The answer to my problem is ____________.
Set A12 ★ Independent Worksheet 2

Operating with Fractions & Whole Numbers

1. Solve each of the story problems below. For each problem:
   - Choose and circle one of the numbers in parentheses, depending on how challenging you want the problem to be.
   - Write an expression to represent your problem.
   - Use numbers, labeled visual models, and/or words to solve the problem and explain your strategy.
   - Complete the sentence below with your solution to the problem.

   a. It takes \( \left( \frac{1}{2}, \frac{1}{3}, \frac{3}{4}, \frac{2}{3} \right) \) of a cup of flour to make a batch of pancakes. I have 4 cups of flour. How many batches of pancakes can I make?

   Expression: ________________________________

   I can make _____ batches of pancakes.

   b. Little Snail can crawl \( \left( \frac{1}{4}, \frac{1}{3}, \frac{3}{4}, \frac{7}{8} \right) \) of a mile a day. How far can he crawl in 5 days if he crawls the same distance each day?

   Expression: ________________________________

   Little snail can crawl ________ miles in 5 days.

(Continued on next page.)
Operating with Fractions & Whole Numbers (cont.)

C  Keiko always takes her water bottle with her when she hikes, and she always drinks \(2 \frac{1}{2}\) cups of water for every mile she hikes. Yesterday, she hiked \(\frac{1}{2}\) a mile. How many cups of water did she drink?

Expression: _______________________________________________

Keiko drank _____ cups of water.

2  Solve each of the multiplication problems below. For each:
- outline a rectangle on the grid that will work for both fractions.
- draw and label the dimensions and area, and write the answer.
- write the problem and answer in words.

<table>
<thead>
<tr>
<th>(\text{ex}) (\frac{2}{3} \times \frac{4}{8} = \frac{8}{24} = \frac{1}{3})</th>
<th>(\frac{1}{24})</th>
<th>(\frac{4}{8})</th>
<th>Two-thirds of 4-eighths is eight twenty-fourths.</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>(\frac{2}{4} \times \frac{3}{5} =)</td>
<td>(\frac{2}{4})</td>
<td>(\frac{3}{5})</td>
</tr>
<tr>
<td>b</td>
<td>(\frac{2}{3} \times \frac{4}{6} =)</td>
<td>(\frac{2}{3})</td>
<td>(\frac{4}{6})</td>
</tr>
<tr>
<td>c</td>
<td>(\frac{3}{4} \times \frac{5}{10} =)</td>
<td>(\frac{3}{4})</td>
<td>(\frac{5}{10})</td>
</tr>
</tbody>
</table>
**Sharing Stories**

**Overview**
Today's activity opens with a short string of division problems. The last problem in the set involves division of a fraction by a whole number. Students review the sharing and grouping interpretations of division, decide which makes more sense in this context, and test their decision by solving and discussing several story problems. The activity concludes with a story designed to help students make generalizations about the effect of dividing a fraction by a whole number.

**Skills & Concepts**
- Divide a unit fraction by a whole number using visual models (CCSS 5.NF.7a)
- Solve story problems involving division of a unit fraction by a whole number (CCSS 5.NF.7c)

**You’ll need**
- Another Division String (page A12.45, run 1 copy for display)
- Fractions on a Geoboard (page A12.46, run 1 copy for display)
- Brownies & Bars (page A12.47, run 1 copy for display)
- Division Posters from Activity 2 (displayed where all the students can see them)
- geoboards and rubber bands (class set, plus 1 board and some bands for display)
- Students’ math journals or 2 pieces of scratch paper per student
- marking pens in black, blue, and yellow
- 1 blue and 1 yellow crayon or colored pencil, per student
- a piece of paper to mask portions of the display

**Instructions for Sharing Stories**
1. Let students know you’re going to start today’s session with another look at the two different interpretations of division the class has been working with lately—sharing and grouping.

2. Display just the top row of the Another Division String master, keeping the rest of the sheet covered for now. Read the combination with the class, and have the students supply the answer as you write it in. Then work with input from the class to write a sharing and a grouping story problem to match, using the theme of dogs and dog biscuits (or any other theme you choose). Review the fact that the quotients mean two different things—in the first case, the size of each group; in the second, the number of groups.

<table>
<thead>
<tr>
<th>Equation</th>
<th>Sharing Problem</th>
<th>Grouping Problem</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{1}{12} \div 4 = \frac{3}{4}$</td>
<td>Brandon had 12 small dog biscuits. He divided them evenly among 4 dogs. How many biscuits did each dog get?</td>
<td>Tiffany made 12 dog biscuits. She put them in bags of 4 to give to her grandparents. How many bags of 4 did Tiffany make?</td>
</tr>
</tbody>
</table>
3. Reveal the next two equations on the master, one by one, and repeat step 2. Then reveal the final expression—\(\frac{1}{2} \div 4\). Read it with the class, and ask students to talk with the people sitting next to them about the answer. After a few moments, invite several volunteers to share their answers with the class. Record all responses without comment, even if some of them are incorrect. (While a few of your students may reason that \(\frac{1}{4}\) of \(\frac{1}{2}\) is \(\frac{1}{8}\) and \(\frac{1}{8}\) of \(\frac{1}{2}\) is \(\frac{1}{16}\), many will likely be confused when presented with the idea of dividing a fraction by a whole number. After all, how can you possibly divide a number less than one by 4?)

4. Resist the temptation to correct student thinking at this point. Instead, move along to posing story problems that represent \(\frac{1}{2} \div 4\). Using the first three grouping problems on the master for reference (as well as the division posters from Session 2 if they seem helpful), solicit agreement from the students that a grouping problem for \(\frac{1}{2} \div 4\) would mean pulling as many groups of 4 as possible out of \(\frac{1}{2}\). Does that make sense?

5. After a minute or two of discussion and debate, table the attempt to write a grouping problem, and explore the possibility of writing a sharing problem to match the combination. This time, solicit agreement from the students that a sharing problem for \(\frac{1}{2} \div 4\) would mean splitting a half of something 4 ways. Is this possible?

While some students may suggest that it would be possible to share half a dog biscuit among 4 dogs, others may argue that the portions would be too small and/or it would be too difficult to split half a biscuit into 4 equal shares. Too, there may be some doubt about the fraction of the biscuit each dog would receive. So, for now, leave the last row on the master blank, promising to come back to it in a few minutes.

6. Acknowledge that it might be difficult to split a half a dog biscuit four ways, and propose instead to model a situation where 4 dogs share a half a pan of cornbread. Explain that the geoboard, which was useful in modeling and solving multiplication with fractions, will prove useful here as well.

7. Display a copy of the Fractions on a Geoboard master while helpers pass out geoboards and bands. Use the master to quickly review with the class some of the fractions that can be formed on the geoboard if the entire board is assigned an area of 1 unit. Have students build each of the regions shown on the master, and report the area as a fraction of the whole. Ask a volunteer or two to explain their thinking before you record the area of each region on the master.

8. When you've completed this brief review, have students clear their geoboards of rubber bands and build Region A again as you do so on your display board. Explain that you'll use this to represent the
half pan of cornbread the 4 dogs are going to share. Then ask the students to use their rubber bands
to divide Region A into 4 equal parts and determine what the size of each part is. As they finish, have
them share and compare their results with the people sitting nearest them.

9. When most students have finished, ask the class to report the size of each part. Record all responses
on the board, and then invite 2 or 3 volunteers to display their geoboards (one at a time) and explain
their thinking to the class.

Max  I just divided the half into 4 parts. I can see that each one of them is the same size as Region C,
so they’re 1⁄8 each.

Emma  I did kind of the same thing, but I divided up the whole pan. I could see there were 8 parts,
so each one must be an 1⁄8.

Demetrius  I split the half into 4 parts, but I put my rubber bands going the other way. Each piece
is still an eighth; it’s just going up and down instead of sideways. Also, I know we’re right because 4
times 1⁄8 is 1⁄2. If you add them up it’s 4⁄8, and that’s the same as 1⁄2.

10. Return to the Division String master, and take a minute to confirm that the model students just built
and explained represents 1⁄2 ÷ 4. Work with students to fill in the answer for the last equation on the
sheet, and record a story problem in the sharing column.

Max  I just divided the half into 4 parts. I can see that each one of them is the same size as Region C,
so they’re 1⁄8 each.

Emma  I did kind of the same thing, but I divided up the whole pan. I could see there were 8 parts,
so each one must be an 1⁄8.

Demetrius  I split the half into 4 parts, but I put my rubber bands going the other way. Each piece
is still an eighth; it’s just going up and down instead of sideways. Also, I know we’re right because 4
times 1⁄8 is 1⁄2. If you add them up it’s 4⁄8, and that’s the same as 1⁄2.

11. Now explain that the geoboard will be a useful way to model and solve some of today’s problems, but
there will be other problems that will require a visual model instead. Have students find the next avail-
able page in their journals, or give them each a piece of scratch paper. Let them know they’ll need a pen-
cil, and crayons or colored pencils in yellow and blue as well.

Have the students each draw a square on their sheet while you do so at the board. Then divide the
square into thirds and shade in the third farthest to the left yellow, and have students do the same.
12. Explain that there was a third of a pan of cornbread leftover from supper the other night, and Alonzo’s mother gave him permission to divide it evenly among their three dogs. Then work with input from the students to write an equation to represent the situation: \( \frac{1}{3} \div 3 \). Give them a few moments to pair-share ideas about what fraction of the pan of cornbread each dog will receive, and then ask students to work with the visual model they’ve made on their paper to determine the answer. (Reassure them that it’s fine to make additional copies of the model if they need to.)

13. Reconvene the class after a minute or two, even if some of the students are still working, or have become stuck. Invite several volunteers to share their thinking with the class by placing their papers on display at the document camera, or drawing on the board.

Josh  I split the third into 3 parts, but I don’t know how much each of those is worth.

Vincent  I did kind of the same thing, but I split the whole pan into 3 parts. I can see there are 9 in the whole pan, so each part of the cornbread must be \( \frac{1}{9} \).

Maya  I agree with you, Vincent. I did the same thing as you, and then I used my blue crayon to color in one of the parts. It’s one out of 9, so that’s a ninth.

14. If no one in your class suggests a strategy similar to Maya’s, model it yourself. Explain or confirm for students that dividing the entire pan, rather than just the fraction of cornbread remaining, makes it possible to determine the size of each portion accurately, as Vincent and Maya have explained above. Then work with students to complete the equation, and reflect on it for a few moments. Does it make logical sense that \( \frac{1}{3} \) divided into 3 equal shares would result in \( \frac{1}{9} \)? Why or why not?
15. Now explain that you have a couple of story problems to pose that might further students' thinking about dividing a fraction by a whole number. Display the Brownies and Bars master as students find the next page in their journals or flip their scratch paper over.

16. Give students a few moments to examine the sheet quietly. Then read and review the instructions and the problems, providing clarification as needed. Be sure to emphasize the requirement that they create a labeled visual model to show and solve the problem they choose.

17. When students understand what to do, have them go to work. Circulate to observe and provide clarification as needed. Encourage children who are struggling to use a geoboard to model and solve the first problem, and then record a visual model and an equation to show their work; the other problem and the two optional challenges require models similar to the one you created for the class.

18. As they finish one of the problems, ask the students to share and compare their work with a classmate, and then solve the other problem. If they finish both, invite them to work on one or both of the challenges.
19. After about 10 minutes, reconvene the class, even if some of the students haven’t finished. Solicit and record all answers to the first problem. Then invite several volunteers to share their work with the class, either at the display, or by replicating their visual model on the board and explaining it to the class. Repeat with the second problem.

Note Both of the story problems feature fractions of baked goods remaining in pans. They were written this way to encourage students to use geoboards or visual models of squares, both of which can be subdivided easily. This model works well for representing and dividing unit or proper fractions by whole numbers. There are other models that can be used, but none that make it quite so easy to determine the fractions that result. For example, some students might try to solve the second problem by modeling the third of the pan of lemon bars that remains, and then dividing that third into 4 equal parts. Problems may emerge when these students try to determine the fractional size of the resulting parts.

Josie I drew the third of a pan and split it up into 4 equal parts, but I have no idea what fraction they are. I guess you could say each of them is a fourth of a third, but that’s kind of weird.

Matt I did the same thing, but I used a circle. I know how to make a third of a circle, but when I divide it into 4 parts, I don’t know what those fractions are.

A model less likely to emerge given the context of today’s problems would be the number line. It’s certainly possible to draw a line, label it 0 at one end and 1 at the other, divide it into thirds, and then subdivide one of the thirds into 4 parts, but unless the student already knows that a fourth of a third is a twelfth, or thinks to subdivide each third into 4 parts, it can be hard to determine the fractional size of each division.

Gregg I thought a number line would work for solving the lemon bar problem, but I didn’t know what size each piece was after I finished making my model.

It’s likely that at least some of your students will use the geoboard or models of squares successfully to find the correct answer to each of the problems. The visual models and answers below are included for your reference. (The challenge problems are optional, but the square models work equally well when dividing unit or proper fractions by whole numbers.)
Problem 1a

\[ \frac{1}{4} \div 4 = \frac{1}{16} \]

Problem 2a

\[ \frac{1}{3} \div 4 = \frac{1}{12} \]

Problem 1b Challenge

\[ \frac{2}{6} \div 4 = \frac{1}{12} \text{ or } \frac{1}{24} \]

Problem 2b Challenge

\[ \frac{2}{5} \div 4 = \frac{1}{20} \text{ or } \frac{1}{10} \]

Explain that you are going to finish up today’s lesson by telling a story that may give students another way to think about dividing fractions and whole numbers. (Even if students grasp the situation clearly at this point, their understandings are likely fragile, and it’s quite possible that a few are still confused by the idea of dividing a fraction by a whole number.)

20. Have students find the next page in their journals or give each of them another a piece of scratch paper, and have them keep their yellow and blue crayons or colored pencils handy. Explain that as you tell the story, they’ll write equations to match, and make visual models to solve the problems that come up. Then start the story, using or adapting the text below as you see fit.

One day, Grandma was baking brownies. Her two grandchildren arrived just as she was cutting up the last pan and packing the brownies into boxes for the bake sale. Grandma said, “I’ve got to run to the store for a few minutes. There’s half a pan of brownies left. Get out the milk, and you can share the half pan between you.” The children’s eyes got wide at the thought of milk and half a pan of Grandma’s best brownies!

21. Stop at this point and ask students to record a division expression to match the situation. Have them hold up their boards or papers, and then call on a volunteer who has correctly recorded \( \frac{1}{2} \div 2 \) to share and explain her thinking. As she does so, record it on the board, and model the situation as the students do so on their boards. When there is general agreement that \( \frac{1}{2} \) divided by 2 is \( \frac{1}{4} \), record the answer on the board. Then continue the story.

Just as the children sat down to enjoy the brownies, there was a knock at the door. It was their cousin, Junior, from next door. They couldn’t leave him out, so they poured a glass of milk for him and re-divided the half-pan of brownies fair and square.
22. Pause again and have students write a second expression under the first to represent the situation. Have them hold up their boards or papers, and then call on a volunteer who has correctly recorded \( \frac{1}{2} \div 3 \) to share and explain her thinking. As she does so, record it on the board, and model the situation as the students do so on their boards. When there is general agreement that \( \frac{1}{2} \) divided by 3 is \( \frac{1}{6} \), record the answer on the board. Then continue the story.

_Just as the children sat down to enjoy the brownies, there was a knock at the door. It was Junior’s little brother. They couldn’t leave him out, so they poured a glass of milk for him and re-divided the half-pan of brownies fair and square._

23. Pause again and take time with the students to model and solve the problem, and record an equation to match. Review the meaning of each of the numbers in the equation with the class. Then discuss the list of equations so far:

- What patterns, if any, can students find in the numbers?
- What is the effect of increasing the number of people who are sharing the brownies?

\[
\begin{align*}
\frac{1}{2} \div 2 &= \frac{1}{4} \\
\frac{1}{2} \div 3 &= \frac{1}{6} \\
\frac{1}{2} \div 4 &= \frac{1}{8}
\end{align*}
\]

24. If time allows, ask students to predict what fraction of the pan each child would get if Junior’s little sister arrived and there were 5 children. And what if Junior’s friend, Maria, came over from across the street and there were 6 children? Press students to explain their predictions as they share.

_Students_ I’m pretty sure it would be \( \frac{1}{10} \) with 5 kids and \( \frac{1}{12} \) with 6 kids.

I agree, Sergio. If you multiply the bottom part of the fraction and the whole number, you get the answer, so \( 2 \times 5 = 10 \) and \( 2 \times 6 = 12 \).

The parts keep getting smaller and smaller.

This is starting to remind me of when we were multiplying fractions. It’s kind of like we’re finding out \( \frac{1}{2} \) of \( \frac{1}{2} \), then \( \frac{1}{3} \) of \( \frac{1}{2} \), then \( \frac{1}{4} \) of \( \frac{1}{2} \).

25. Let students know you’ll return to the problem of dividing fractions by whole numbers in the next activity.
Another Division String

<table>
<thead>
<tr>
<th>Equation</th>
<th>Sharing Problem</th>
<th>Grouping Problem</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 12 ÷ 4 = _______</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2 8 ÷ 4 = _______</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3 4 ÷ 4 = _______</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4 ½ ÷ 4 = _______</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Fractions on a Geoboard

If the biggest square on the geoboard has an area of 1 unit, what is the area of each lettered region? Label the area of each region below.

Area of Region A = _____ square unit
Area of Region B = _____ square unit
Area of Region C = _____ square unit
Area of Region D = _____ square unit
Area of Region E = _____ square unit
Brownies & Bars

Choose one of the two story problems below and copy it into your journal (or onto your piece of paper). Then:

• create a labeled visual model to represent and solve the problem.
• write an equation to match.

You will have about 10 minutes to work. If you finish one problem, do the other, or solve the challenge version of the problem you started with.

1a There is \( \frac{1}{4} \) of a pan of brownies left. Jake and his three brothers are going to share it equally. What fraction of the pan of brownies will each of the 4 boys get?

Equation: ________________________________

1b Challenge What if there was \( \frac{2}{6} \) of a pan of brownies left? Then what fraction of the pan of brownies would each boy get if they divided it equally among themselves?

Equation: ________________________________

2a There is \( \frac{1}{3} \) of a pan of lemon bars left. Sara and her three cousins are going to share it equally. What fraction of the pan of lemon bars will each of the 4 girls get?

Equation: ________________________________

2b Challenge What if there was \( \frac{2}{5} \) of a pan of lemon bars left? Then what fraction of the pan of lemon bars would each girl get if they divided it equally among themselves?

Equation: ________________________________
Set A12 ★ Activity 6

Dividing a Fraction by a Whole Number

Overview
After a short review of the sharing and grouping models of division, students work independently to solve two story problems. The teacher reconvenes the class and works with the students to record several strategies for dividing unit fractions by whole numbers. Students then complete a related assignment.

Skills & Concepts
★ Divide a unit fraction by a whole number using visual models (CCSS 5.NF.7b)
★ Create a story context for dividing a unit fraction by a whole number (CCSS 5.NF.7b)
★ Solve story problems involving division of a unit fraction by a whole number (CCSS 5.NF.7c)

You’ll need
★ Rope & Candy Problems (page A12.54, run 1 copy for display)
★ More Do-It-Yourself Story Problems (pages A.12.55–A12.57, run 1 copy for display and a class set on paper)
★ Division Posters from Activity 2 (displayed where all the students can see them)
★ Students’ math journals or 1 piece of scratch paper per student
★ 2 pieces of chart paper
★ marking pens

Instructions for Dividing a Fraction by a Whole Number
1. Open today’s instruction by letting students know that you’re going to start with a short review of the two different interpretations of division the class has been using lately—sharing and grouping.

2. Draw students’ attention to the two posters created by the class during Activity 2. Give students a few moments to study the posters quietly. Then write $4 \div 2$ on the board and ask them to brainstorm, first in pairs and then as a whole group, a sharing and a grouping story problem to match. Record an example of both types of story problems on the board, along with the solution to each. Note with students that the quotient means two different things: the size of the group in the case of the sharing problem, and the number of groups in the grouping problem.

<table>
<thead>
<tr>
<th>Sharing Story Problem</th>
<th>Grouping Story Problem</th>
</tr>
</thead>
<tbody>
<tr>
<td>I had 4 horse posters. I split them evenly with my best friend. How many posters did we each get?</td>
<td>I had 4 sports posters. I pinned up 2 of them on each wall in my room until I ran out. How many walls got posters?</td>
</tr>
<tr>
<td>Answer: 2 posters</td>
<td>Answer: 2 walls</td>
</tr>
</tbody>
</table>
3. Next, write \( \frac{1}{4} \div 2 \) on the board and read it with the class. Then go back and read it to the class as a sharing and then a grouping situation. Which interpretation makes sense?

**Teacher**  Okay, I think we all agree that this expression says \( \frac{1}{4} \) divided by 2. I'm going to phrase it in two different ways now—sharing first, and then grouping. Think privately about which version makes more sense, and then we'll talk.

One-fourth of something shared evenly between 2 people.
The number of groups of 2 you can get out of \( \frac{1}{4} \).

**Students**  You can't get any groups of 2 out of \( \frac{1}{4} \). Two is way bigger than \( \frac{1}{4} \)!
I think you have to use the sharing way.
But if you had \( \frac{1}{4} \) of something really big, you could give it to lots of pairs of people.
I respectfully disagree, Jason. You can't say that 2 is smaller than \( \frac{1}{4} \) all of a sudden. Two is, like... 8 times bigger than one-fourth because there are \( \frac{1}{4} \) in 1.
Sharing works, though. You can share a fourth of a cookie or a sandwich with someone. You don't get very much, but you can do it.
If you cut a \( \frac{1}{4} \) in \( \frac{1}{2} \) to share it with someone else, it's \( \frac{1}{8} \), because there are \( \frac{2}{8} \) in a \( \frac{1}{4} \).

4. Even if a few students are still somewhat confused by the situation, confirm what many probably understand by now—when you divide a unit fraction by a whole number, the sharing interpretation makes better sense, than the grouping interpretation. In fact, \( \frac{1}{4} \div 2 \) requires the problem-solver to divide one-fourth into two equal parts and then determine the size of each fractional part that results.

5. Have students brainstorm, first in pairs and then as a whole group, a sharing story problem to match \( \frac{1}{4} \div 2 \). Record one of their ideas on the board, and work with the class to solve the problem, using a visual model similar to the ones you introduced last session. Encourage students who are still struggling with the concept of dividing a fraction by a whole number to model the situation on a geoboard.

6. Next, display the Rope & Candy Problems master as students get out their math journals or scratch paper.

7. Give students a few moments to examine the sheet quietly. Then read and review the instructions and the problems, providing clarification as needed without telling the students how to solve either of
the problems. Be sure to emphasize the requirement that they create a labeled visual model to show and solve the problem they choose.

8. When students understand what to do, have them go to work. Circulate to observe and provide clarification if necessary, but avoid the temptation to tell students how to model or solve the problems. (Encourage students who may be getting frustrated at this point to see if they can model and solve the second problem on a geoboard, and then create a labeled visual model to show their work.)

9. As they finish one of the problems, ask the students to share and compare their work with a classmate, and then either solve the other problem or work on the challenge associated with the problem they chose first.

10. After about 10 minutes, reconvene the class, even if some of the students haven't finished. Solicit and record all answers to the first problem. Then invite several volunteers to share their work with the class, either at the display, or by replicating their model on the board and explaining it to the class. Repeat this process with the second problem.

After each volunteer has shared, work with input from the class to replicate the model and summarize the strategy used to solve the problem on a piece of chart paper.
Models and Strategies for Dividing a Unit Fraction by a Whole Number

- Draw a number line up to 1. Mark it at \( \frac{1}{2} \), and then divide each half into 3 parts. That way, you can tell how many there are in 1, and what size they each are.

\[
\begin{array}{cccc}
0 & \frac{1}{6} & \frac{1}{4} & \frac{1}{6} \\
\frac{1}{2} \text{ yard} & \frac{1}{6} & \frac{1}{4} & \frac{1}{6} \\
1 & \frac{1}{6} & \frac{1}{4} & \frac{1}{6}
\end{array}
\]

If you split each half into 3 parts, there are 6 in all, so each part must be \( \frac{1}{6} \) of a yard.

- Draw a square to stand for the whole yard. Divide it in half one way, and in 3 parts the other way. Then you can see how big each part is when you divide \( \frac{1}{2} \) into 3 parts. It’s \( \frac{1}{6} \), so each part of the rope must be \( \frac{1}{6} \) of a yard.

\[
\begin{array}{c}
\frac{1}{2} \\
\frac{1}{6} \\
\frac{1}{6} \\
\frac{1}{6} \\
\frac{1}{6} \\
\frac{1}{6}
\end{array}
\]

Divided by 3

Note: The strategies shown on the charts above represent some of the ways fifth graders are likely to deal with dividing fractions by whole numbers in the context of selected story problems. Some of these strategies are more efficient than others, and it’s entirely possible that your students will devise methods that are not shown above. It should be stressed that you are not responsible for teaching any of these strategies directly. The goal of your instruction at this point should be to provide students with opportunities to make sense of situations that involve division of unit fractions by whole numbers, to ask that they bring perseverance, a willingness to take risks, and the knowledge they already possess about division and fractions to bear on problems that require them to stretch a little (or a lot, in some cases).

11. Give students each a copy of the More Do-It-Yourself Story Problems and display the masters for everyone to see. Review and discuss the tasks with the class. Give students the option of working on these sheets independently, in pairs, or with you. Encourage students to make use of the strategies just shared by their classmates.
12. As students finish, have them share and compare their work with a classmate. If they have selected different divisors or dividends for each problem, challenge them to be sure each other’s answers are correct. Have them check each other’s story problems to be sure they make sense and have been solved correctly.

If students need more time to complete the assignment, ask them to finish it as homework or as part of their morning seatwork the following day before you conduct Activity 7.

**Note**  Save the strategy posters from today’s activity for use in Activity 7.

---

**INDEPENDENT WORKSHEET**

See Set A12 Independent Worksheet 3 on page A12.79 for more practice with dividing a fraction by a whole number, using the relationship between multiplication and division to check answers, and multiplying fractions and whole numbers.
Rope & Candy Problems

Choose one of the two story problems below and copy it into your journal or paper. Then:
• create a labeled visual model to represent and solve the problem.
• write an equation to match.

You will have about 10 minutes to work. If you finish one problem, do the other, or solve the challenge version of the problem you started with.

1a John has \( \frac{1}{2} \) of a yard of rope. He wants to cut the rope into 3 equal pieces. What fraction of a yard will each piece of rope be?

Equation: _______________________________________________

1b Challenge How many inches long will each piece of rope be?

Equation: _______________________________________________

2a Lori had \( \frac{1}{4} \) of a bag of candy and shared it with 3 friends. What fraction of the bag of candy did each of the 4 children get?

Equation: _______________________________________________

2b Challenge There were 64 pieces of candy in the whole bag. How many pieces of candy did each friend get?

Equation: _______________________________________________
More Do-It-Yourself Story Problems  Page 1 of 3

1  Solve each of the story problems below. For each problem:
• Choose and circle one of the numbers in parentheses.
• Write an expression to represent your problem.
• Use numbers, labeled visual model, and/or words to solve the problem.
• Complete the sentence below with your solution to the problem

a  (Two, three, four) dinner guests shared (\(\frac{1}{2}\), \(\frac{1}{3}\), \(\frac{2}{3}\), \(\frac{3}{4}\)) of a pan of cornbread. What fraction of the cornbread did each guest get?

Expression: _______________________________________________

Each guest got ______ pan of cornbread.

b  Five cousins shared (\(\frac{1}{2}\), \(\frac{1}{3}\), \(\frac{1}{4}\), \(\frac{3}{4}\)) of a birthday cake. What fraction of the cake did each cousin get?

Expression: _______________________________________________

Each cousin got ______ of the birthday cake.

(Continued on next page.)
More Do-It-Yourself Story Problems  Page 2 of 3

C  Sara had $\left(\frac{1}{2}, \frac{1}{3}, \frac{2}{3}\right)$ of a cup of grated cheese. She divided it equally between 4 salads. What fraction of a cup of cheese did each salad get?

Expression: _______________________________________________

There was _____ a cup on each salad.

d  Mr. Brown had $\frac{1}{8}$ of a pack of paper. He divided the pack equally among equally among 3 students. What fraction of the pack of paper did each student get?

Expression: _______________________________________________

Each student got _____ a pack of paper.

(Continued on next page.)
There were 504 sheets of paper in the whole pack. How many sheets of paper did each of the three students get?

Expression: _______________________________________________

Each student got ____ sheets of paper.

Choose one of the expressions below and circle your choice. Write a story problem about the expression you circled. Then solve your own problem.

\[ \frac{1}{2} \div 2 \quad \frac{1}{2} \div 6 \quad \frac{1}{3} \div 4 \quad \frac{1}{8} \div 4 \]

My story problem is:

My work:

The answer to my problem is:_______
More Fractions & Whole Numbers

1. Solve each of the story problems below. For each problem:
   • Choose and circle one of the numbers in parentheses, depending on how challenging you want the problem to be.
   • Write an expression to represent your problem.
   • Use numbers, labeled visual models, and/or words to solve the problem and explain your strategy. (Someone should be able to read your paper and tell how you solved each problem without talking to you to find out.)
   • Complete the sentence below with your solution to the problem.

   a. Mrs. Alvarez had (\( \frac{1}{5} \), \( \frac{1}{8} \), \( \frac{3}{8} \), \( \frac{2}{3} \)) of a box of pencils. She divided the box equally among (3, 4, 5) students. What fraction of the box of pencils did each student get?

      Expression: _______________________________________________

      Each student got _____ of a box of pencils.

   b. Sara has a rug in her bedroom that is (2, 3, 4 feet) by (2 \( \frac{1}{2} \), 2 \( \frac{3}{4} \), 3 \( \frac{3}{8} \) feet). What is the area of Sara’s rug?

      Expression: _______________________________________________

      The area of Sara’s rug is _______ feet.

      (Continued on next page.)
More Fractions & Whole Numbers (cont.)

C It takes \(4 \frac{1}{2}, 4 \frac{3}{4}, 4 \frac{1}{4}\) feet of craft lace to make a short lanyard for a keychain. John wants to make a lanyard for each of his (5, 6, 7) aunts and uncles. How many feet of craft lace will he need in all?

Expression: _______________________________________________

John will need _____ feet of craft lace.

2 Use multiplication to check your answer for each of the division problems below.

<table>
<thead>
<tr>
<th>(100 \div 4 = \underline{25})</th>
<th>(\frac{1}{2} \div 2 = \underline{\frac{1}{4}})</th>
</tr>
</thead>
<tbody>
<tr>
<td>I know this is correct because (25 \times 4 = 100)</td>
<td>I know this is correct because (\frac{1}{4} \times 2 = \frac{1}{2})</td>
</tr>
<tr>
<td>a (\frac{1}{2} \div 4 = _____)</td>
<td>b (\frac{1}{4} \div 2 = _____)</td>
</tr>
<tr>
<td>c (\frac{1}{3} \div 2 = _____)</td>
<td>d (\frac{1}{4} \div 4 = _____)</td>
</tr>
</tbody>
</table>

3 Maria says that dividing \(\frac{1}{2}\) by 3 is the same as multiplying \(\frac{1}{2}\) by \(\frac{1}{3}\). Do you agree with her? Why or why not? Use numbers, labeled models, and/or words to explain your thinking.
The Division Poster Project

Overview
Today, students are asked to deal with situations that involve dividing whole numbers by unit fractions and unit fractions by whole numbers within the same instructional period. To start, the class examines $4 \div \frac{1}{3}$ and $\frac{1}{3} \div 4$, thinking about which of the two interpretations of division makes more sense with regard to solving each problem. Students then generate and solve story problems to match each expression. Finally, students work in pairs to create posters representing their understandings of dividing whole numbers by unit fractions and unit fractions by whole numbers.

Note  Plan to take two math periods for this project.

Skills & Concepts
- Divide a whole number by a unit fraction using visual models (CCSS 5.NF.7a)
- Create a story context for dividing a whole number by a unit fraction (CCSS 5.NF.7a)
- Divide a unit fraction by a whole number using visual models (CCSS 5.NF.7b)
- Create a story context for dividing a unit fraction by a whole number (CCSS 5.NF.7b)
- Solve story problems involving division of a whole number by a unit fraction, and a unit fraction by a whole number (CCSS 5.NF.7c)

You'll need
- Division Poster Project (page A12.64, run 1 copy for display)
- Division Posters from Activity 2 (displayed where all the students can see them)
- Strategy Posters from Activities 4 and 6 (displayed where all the students can see them)
- 2 pieces of unlined chart paper (see Advance Preparation)
- marking pens
- 1 piece of scratch paper per student pair
- 2" × 18" drawing paper, 1 sheet per student pair
- additional scratch paper (optional, see note at the end of the activity)
- crayons, colored pencils, and/or markers for student use
- students’ name tags or some other way to draw children’s names at random

Advance Preparation  Write $4 \div \frac{1}{3}$ at the top of one of the pieces of unlined chart paper. Write $\frac{1}{3} \div 4$ at the top of the other piece.

Instructions for The Division Poster Project
1. Open today’s instruction by letting students know that they’re going to work in pairs to create posters to show what they’ve learned about dividing whole numbers and fractions a little later in the session, but first you’re going to do some review work together.
2. Draw students’ attention to the two pieces of chart paper you’ve prepared for today’s activity. Read the expressions at the top of each sheet with the class, and ask students to share, first in pairs and then as a whole group, which interpretation of division makes better sense for each. (Encourage students to refer to the Division Posters the class made during Session 2 and/or the Strategy Posters from Sessions 4 and 6 if those seem helpful.)

   **Natasha** Okay, on that first one, it says 4 divided by a third. Sharing doesn’t make any sense with that one, because how can you share 4 things with a third of a person?

   **Eric** I think sharing makes sense with the other one, though. You can take \( \frac{1}{3} \) of something and share it with 4 people, like if you have \( \frac{1}{3} \) of a pizza or something like that.

   **Amber** On that first one, I think you have to use grouping. You can take groups of one third out of 4. There are 3 thirds in 1, right? So there must be 12 thirds in 4.

   **Carlos** But grouping doesn’t make any sense with \( \frac{1}{3} \div 4 \), because there aren’t any groups of 4 in \( \frac{1}{3} \). One-third is smaller than 4 by a long shot.

3. When there is general consensus that \( 4 \div \frac{1}{3} \) makes the most sense when interpreted as a situation that requires grouping, while \( \frac{1}{3} \div 4 \) makes more sense in the context of sharing, work with students to record that information below each expression on the chart paper. Have them help you write out a verbal “translation” of each expression as well (see illustration below).

4. Next, work with the class to generate and record a story problem for each expression. (Don’t take a lot of time with this step, because students will soon be working in pairs to devise their own story problems for the two expressions.)

   \[
   \begin{align*}
   4 \div \frac{1}{3} & \\
   \text{Grouping} & \\
   \text{How many groups of } \frac{1}{3} \text{ are there in 4?} & \\
   \text{How many thirds are there in 4?} & \\
   \text{Mrs. Brown had 4 apples. She cut each apple into thirds. How many thirds did she make in all?} & \\
   \end{align*}
   \]

   \[
   \begin{align*}
   \frac{1}{3} \div 4 & \\
   \text{Sharing} & \\
   \text{If 4 people share } \frac{1}{3} \text{ of a _____, what fraction of the _____ will they each get?} & \\
   \text{There was } \frac{1}{3} \text{ of a pizza leftover from dinner. Mom said the 4 kids could split it among themselves for a snack the next day.} & \\
   \end{align*}
   \]

5. Have students pair up, or assign partners. Give each pair a piece of scratch paper, and have them write their own story problems, one for \( 4 \div \frac{1}{3} \) and one for \( \frac{1}{3} \div 4 \). (Suggest that they write each expression and a matching story problem at the top of one side of the scratch paper.)

6. As each pair finishes writing their two story problems, have them meet with another pair, trade papers, and solve each other’s problems. Let them know that they’ll need to show and explain their work in writing, using numbers, labeled models, and words, because each pair should be able to understand how the other pair solved their problems by reading their responses without any additional explanation.
Teacher If you, Marcus and Andrew, solve the problems I wrote with Andrea, she and I should be able to see exactly how you solved them just by reading your work—without having to ask you questions about it. That means you’re going to have to show and explain each step you take. The answer by itself doesn’t explain how the problem was solved.

7. When students have had adequate time to complete the assigned tasks, reconvene the class. Call on students at random to ensure maximum engagement, asking each to read one of the story problems he or she wrote. Have students identify the matching expression, and work with their input to record the context or situation on the appropriate chart.

Teacher I have all your name tags. I am going to mix them up and pull them out at random, one by one. If I pull your name, you get to read one of the story problems you wrote without telling us which expression it was for. We have to figure it out and listen carefully enough to your problem to be able to summarize the context or situation and record it on the correct chart. When we finish, we’ll have a whole list of ideas for writing more story problems. Maya, I just pulled your name. What do you have for us?

Maya There was a third of a pan of brownies on the counter. My mom said my 3 sisters and I could share it. How much did we each get?

Teacher Thumbs up if you know which expression Maya’s story problem matches. Okay, you can all tell me at once—ready?

Students $\frac{1}{3} \div 4$

Teacher Is that correct, Maya?

Maya Yep!

Teacher So, Maya has 4 kids sharing a third of a pan of brownies. I’ll record that on the correct piece of chart paper here. Ernest, you’re up next.

Here’s how the two charts might look after you have called on 8–10 students, which should be adequate.
Activity 7  The Division Poster Project (cont.)

Set A12 Number & Operations: Dividing Fractions & Whole Numbers

4 ÷ \( \frac{1}{3} \)

Grouping
How many groups of \( \frac{1}{3} \) are there in 4?
How many thirds are there in 4?

Mrs. Brown had 4 apples. She cut each apple into thirds. How many thirds did she make in all?

Contexts:
- 4 giant cookies, each cut into thirds—how many thirds?
- 4 mini-pizzas, each cut into thirds
- 4 pieces of colored paper, each cut into thirds
- 4 yards of cloth, each cut into thirds to make flags

\( \frac{1}{3} \) ÷ 4

Sharing
If 4 people share \( \frac{1}{3} \) of a ______, what fraction of the _____ will they each get?

There was \( \frac{1}{3} \) of a pizza leftover from dinner. Mom said the 4 kids could split it among themselves for a snack the next day.

Contexts:
- 4 kids sharing \( \frac{1}{3} \) of a pan of brownies
- 4 dogs sharing \( \frac{1}{3} \) of a pan of cornbread
- 4 friends sharing \( \frac{1}{3} \) of a giant pizza
- 4 sisters sharing \( \frac{1}{3} \) of a box of candy
- \( \frac{1}{3} \) of a yard of ribbon cut into 4 equal pieces

8. Finally, display the Division Project Poster master. Give students a minute to read it to themselves. Then read it with the class and clarify the assignment as needed.

---

**Division Poster Project**

You and your partner are going to work together to make a poster showing what you have learned about dividing whole numbers and fractions. Follow these instructions:

1. **Choose one of the pairs of expressions listed below.**
   - \( 4 \div \frac{1}{3} \)  \( \frac{1}{3} \div 4 \)
   - \( 5 \div \frac{1}{4} \)  \( \frac{1}{4} \div 5 \)
   - \( 6 \div \frac{1}{3} \)  \( \frac{1}{3} \div 6 \)

2. **Challenge:**
   - \( 6 \div \frac{1}{4} \)  \( \frac{1}{4} \div 6 \)

2. Fold a piece of drawing paper in half. On each half:
   - Write one of the expressions from the pair you picked.
   - Indicate whether the expression should be solved using the sharing or the grouping interpretation of division, and explain why.
   - Write a story problem to match the expression.
   - Solve your story problem. Show all your work. Your solution must include a clearly labeled visual model.
9. When students understand what to do, give each pair a piece of $12" \times 18"$ drawing paper and let them go to work. Remind them that they can use any of the story problem contexts or situations on the charts you made with the class today, or think of new ones if they prefer.

---

**Note**  You may want to have students draft their story problems and solutions on scratch paper, and then check them with another student pair for accuracy, before they commit their work to drawing paper.

---

**Extensions**

- When student pairs have completed their posters, display them in the classroom or the hallway. If you choose to display them in the hallway where they can be viewed by students, teachers, and parents, you might also post a copy of the Division Poster Project master, or perhaps a shorter explanation of the project you (or you and the students) write, so others can appreciate what went into these posters.

- Once the posters are up, have your students take a gallery walk, moving through the display in pairs to examine and admire their classmates’ work.
Division Poster Project

You and your partner are going to work together to make a poster showing what you have learned about dividing whole numbers and fractions. Follow these instructions:

1. Choose one of the pairs of expressions listed below.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$4 \div \frac{1}{2}$</td>
<td>$\frac{1}{2} \div 4$</td>
</tr>
<tr>
<td>$5 \div \frac{1}{4}$</td>
<td>$\frac{1}{4} \div 5$</td>
</tr>
<tr>
<td>$8 \div \frac{1}{4}$</td>
<td>$\frac{1}{4} \div 8$</td>
</tr>
<tr>
<td>$6 \div \frac{1}{3}$</td>
<td>$\frac{1}{3} \div 6$</td>
</tr>
</tbody>
</table>

**CHALLENGE**

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$6 \div \frac{2}{3}$</td>
<td>$\frac{2}{3} \div 6$</td>
</tr>
<tr>
<td>$6 \div \frac{3}{4}$</td>
<td>$\frac{3}{4} \div 6$</td>
</tr>
</tbody>
</table>

2. Fold a piece of drawing paper in half. On each half:
   - write one of the expressions from the pair you picked.
   - tell whether the expression should be solved using the sharing or the grouping interpretation of division, and explain why.
   - write a story problem to match the expression.
   - solve your story problem. Show all your work. Your solution must include a clearly labeled visual model.
Set A12 ★ Activity 8

ACTIVITY

Dividing Fractions & Whole Numbers Post-Assessment

Overview
The pre-assessment given in Activity 1 is re-administered in somewhat different form during this activity, and includes skills addressed in Supplement Set A9 as well as Supplement Set A12. Students’ work on the post-assessment will provide information about what they have learned, as well as the areas in which they need continued support.

Skills & Concepts
★ Multiply a whole number or a fraction by a fraction (CCSS 5.NF.4a)
★ Solve story problems involving multiplying a whole number or a fraction by a fraction (CCSS 5.NF.4a)
★ Solve story problems involving multiplication of fractions and mixed numbers (CCSS 5.NF.6)
★ Divide a whole number by a unit fraction (CCSS 5.NF.7a)
★ Create a story context for dividing a whole number by a unit fraction (CCSS 5.NF.7a)
★ Divide a unit fraction by a whole number (CCSS 5.NF.7b)
★ Create a story context for dividing a unit fraction by a whole number (CCSS 5.NF.7b)
★ Solve story problems involving division of a whole number by a unit fraction, and a unit fraction by a whole number (5.NF.7c)

You’ll need
★ Operating with Fractions & Whole Numbers Post-Assessment (pages A12.67–A12.70, run a class set plus a copy for display)
★ Grid Paper (page A12.71, copies as needed)
★ Operating with Fractions & Whole Numbers Post-Assessment Class Checklist (optional, pages A12.72 and A12.73, run 1 or 2 copies)
★ Geoboards and bands available for students who want to use them

Instructions for Dividing Fractions & Whole Numbers Post-Assessment
1. Give each student a copy of the post-assessment and then read and review the tasks with the class. Have students write their names on their papers and circle each “doing” word as you read through the items together.
2. Before students start to work, be sure they understand that they have to use numbers, labeled models, and/or words to show their work and/or explain their thinking for problems 1–3; the answers alone will not be adequate. Remind them that they need to write a story problem to match the expression in problems 4 and 5, but they don’t need to solve the problems. Also, alert them to the fact that this assessment includes multiplication as well as division situations because it’s important to be able to tell when each operation is applied.

Let them know that they can use geoboards and bands, and/or grid paper to help solve any of the problems on the assessment, and make sure they understand how to access these materials.

3. Remind students that you are available to re-read any of the directions or problems for them while they work. Advise them to complete the items they find easiest and most familiar first, even if that means skipping around and then returning to the questions they find more challenging.

4. If you plan to score this assessment as suggested on the Operating with Fractions & Whole Numbers Post-Assessment Class Checklist, let students know how you will be scoring their papers. In some of the problems, they will be given a point for the answer and a point for showing their work. Story problems will be scored on a 3-point basis as follows:
   - 1 point for writing an expression that accurately represents the story problem
   - 1 point for using a strategy that could lead to the correct answer.
   - 1 point for the correct answer, clearly stated.

While it may seem to create test anxiety, we find it is helpful to share expectations with students before they begin.

5. Give students the rest of the period to complete the assessment. Make sure your students understand what they are expected to do when they complete the assessment and where you want them to place their finished papers.

Note: In addition to scoring students post-assessments as suggested on the Post-Assessment Class Checklist, you may find it helpful to compare them to students’ pre-assessments. Although some students may not score particularly well on the post-assessment, you may find they’ve made progress since the beginning of this supplement set.
1. Solve each of the five story problems below. For each problem:
   • Write an expression to represent the problem.
   • Solve the problem. Show your work with labeled visual models, numbers, and/or words.
   • Complete the sentence below with your solution to the problem.

a. It takes $\frac{1}{2}$ of a cup of flour to make a batch of pancakes. Curtis has 4 cups of flour. How many batches of pancakes can he make?

Expression: _______________________________________________

Curtis can make ________ batches of pancakes.

b. The fifth graders are painting the bookshelves in their classroom. It takes $\frac{3}{4}$ of a quart of paint to paint each bookshelf. There are 8 bookshelves in the room. How many quarts of paint will the kids need to paint all 8 bookshelves?

Expression: _______________________________________________

The kids will need ________ quarts of paint to paint all 8 bookshelves.

(Continued on next page.)
C  There is 1/3 of a pan of cornbread left. Four children are going to share it equally. What fraction of the whole pan of cornbread will each child get?
Expression: ____________________________________________

Each child will get ________________________________ of the pan of cornbread.

D  There was 1/2 of a cake left over from Hannah’s birthday party. When she and her sister came home from school the next day, they ate 2/3 of the leftover cake for a snack. How much of the whole cake did the girls have for a snack?
Expression: ____________________________________________

The girls had __________ of the whole cake for a snack.

(Continued on next page.)
The Ruiz family is going to build a raised garden bed for planting flowers in. The bed will be $\frac{3}{4}$ meter wide and $3 \frac{1}{2}$ meters long. What will the area of the raised bed be when it is finished?

Expression: _______________________________________________

The area of the raised flower bed will be ____________ square meters.

2 Cory says that $6 \div \frac{1}{2}$ means the same thing as $\frac{1}{2}$ of 6, so the answer is 3. Do you agree with him? Why or why not? Use numbers, labeled models, and/or words to explain your thinking.

3 Jade says she knows that $\frac{1}{4} \div 4 = \frac{1}{16}$ because $\frac{1}{16} \times 4 = \frac{1}{4}$. Is she correct? Explain why or why not.
4 Write a story problem to represent the expression in the box below. Then solve your own problem. Show your work with labeled models, numbers, and/or words, and write the answer on the line provided.

\[
\frac{1}{2} \div 3
\]

My story problem:

My work:

The answer to my problem is __________.

5 Write a story problem to represent the expression in the box below. Then solve your own problem. Show your work with labeled models, numbers, and/or words, and write the answer on the line provided.

\[
6 \div \frac{1}{3}
\]

My story problem:

My work:

The answer to my problem is __________.
Grid Paper
<table>
<thead>
<tr>
<th>Item and Correct Answer</th>
<th>CCSS</th>
<th>Points Possible</th>
<th>Student Names</th>
</tr>
</thead>
<tbody>
<tr>
<td>1a Write an expression to represent a story problem that involves dividing a whole number by a unit fraction. Solve the problem; show work. Expression: $4 \div \frac{1}{2}$ Answer: 8 batches of pancakes; student work will vary</td>
<td>5.NF.7b 5.NF.7c</td>
<td>3 pts.</td>
<td></td>
</tr>
<tr>
<td>1b Write an expression to represent a story problem that involves multiplying a whole number by a fraction. Solve the problem; show work. Expression: $8 \times \frac{3}{4}$ OR $\frac{3}{4} \times 8$ Answer: 6 quarts; student work will vary</td>
<td>5.NF.4a</td>
<td>3 pts.</td>
<td></td>
</tr>
<tr>
<td>1c Write an expression to represent a story problem that involves dividing a unit fraction by a whole number. Solve the problem; show work. Expression: $\frac{1}{2} \div 4$ Answer: $\frac{1}{8}$ the pan of cornbread; student work will vary</td>
<td>5.NF.7a 5.NF.7c</td>
<td>3 pts.</td>
<td></td>
</tr>
<tr>
<td>1d Write an expression to represent a story problem that involves multiplying a fraction by a fraction. Solve the problem; show work. Expression: $\frac{1}{2} \times \frac{2}{3}$ Answer: $\frac{1}{3}$ of the whole cake; student work will vary</td>
<td>5.NF.4a</td>
<td>3 pts.</td>
<td></td>
</tr>
<tr>
<td>1e Write an expression to represent a story problem that involves multiplying a fraction by a mixed number. Solve the problem; show work. Expression: $\frac{3}{4} \times 3\frac{1}{2}$ Answer: 2 $\frac{3}{4}$ square meters; student work will vary</td>
<td>5.NF.6</td>
<td>3 pts.</td>
<td></td>
</tr>
<tr>
<td>2 Interpret division of a whole number by a unit fraction. Answer: No; students' explanations will vary. Example: I don't agree because $6 \div \frac{1}{2}$ is not the same thing as half of 6. It means how many halves in 6. Since there are 2 halves in 1, there are 12 halves in 6.</td>
<td>5.NF.7b</td>
<td>2 pts.</td>
<td></td>
</tr>
<tr>
<td>Item and Correct Answer</td>
<td>CCSS</td>
<td>Points Possible</td>
<td></td>
</tr>
<tr>
<td>-------------------------</td>
<td>------</td>
<td>-----------------</td>
<td></td>
</tr>
<tr>
<td>3 Recognize the relationship between division and multiplication</td>
<td>5.NF.7a</td>
<td>2 pts.</td>
<td></td>
</tr>
<tr>
<td>Answer: Yes; students' explanations will vary. Example: I agree because $4 \times \frac{1}{16}$ is a fourth. You can show that by adding $1 \frac{1}{16}$ four times. You get $\frac{4}{16}$, which is a fourth, so $\frac{1}{16}$ divided by 4 must be $\frac{1}{16}$.</td>
<td>5.NF.7a</td>
<td>1 pt. for correct answer</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>1 pt. for giving a viable explanation</td>
<td></td>
</tr>
<tr>
<td>4 Write and solve a story problem to represent the expression $\frac{1}{2} \div 3$; show work.</td>
<td>5.NF.7a</td>
<td>3 pts.</td>
<td></td>
</tr>
<tr>
<td>Answer: $\frac{1}{6}$; Students' story problems and work will vary. Sample problem: I had half a sandwich. I gave it to my 3 friends to share equally. How much of the sandwich did each friend get?</td>
<td>5.NF.7b</td>
<td>1 pt. for a story problem that accurately represents the expression</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>1 pt. for showing work that involves the use of a strategy that could lead to the right answer</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>1 pt for the correct answer</td>
<td></td>
</tr>
<tr>
<td>5 Write and solve a story problem to represent the expression $6 \div \frac{1}{3}$; show work.</td>
<td>5.NF.7b</td>
<td>3 pts.</td>
<td></td>
</tr>
<tr>
<td>Answer: 18; Students' story problems and work will vary. Sample Problem: I had 6 apples. I cut each apple into thirds. How many thirds did I get in all?</td>
<td>5.NF.7b</td>
<td>1 pt. for a story problem that accurately represents the expression</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>1 pt. for showing work that involves the use of a strategy that could lead to the right answer</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>1 pt for the correct answer</td>
<td></td>
</tr>
</tbody>
</table>

Total Score/Level of Proficiency*

25 pts

* Meeting Standard: 19–25 points (75–100% correct)  Approaching Standard: 13–18 points (50–74% correct)  Strategic: 7–12 points (25–49% correct)  Intensive: 6 points or fewer (24% or less correct)
GRADE 5 – UNIT 7

CCSS SUPPLEMENT ACTIVITIES & INDEPENDENT WORKSHEETS

Set A10: Number & Operations: Integers
Activity 1: Introducing Integers .................................................. A10.1
Ind. Worksheet 1: Negative & Positive Temperature .................. A10.29
Activity 2: Integer Tug O’ War ................................................. A10.13
Ind. Worksheet 2: Temperature & Elevation Riddles ................. A10.31
Activity 3: 4-Quadrant Battleship ............................................. A10.23
Ind. Worksheet 3: Shapes on a 4-Quadrant Grid ....................... A10.33

Set B1: Algebra: Diagrams & Equations
Activity 1: The Carnival .......................................................... B1.1
Ind. Worksheet 1: Padre’s Pizza ............................................... B1.7
Ind. Worksheet 2: Choosing Equations & Diagrams .................. B1.11

Set A4: Number & Operations: Long Division
Activity 1: Introducing the Standard Algorithm ......................... A4.1
Activity 2: Extending the Standard Algorithm ......................... A4.11
Introducing Integers

Overview
Students discuss the definitions of counting numbers, whole numbers, and integers. Then the class plays a game designed to help students understand integers by relating them to elevation: above sea level, at sea level, and below sea level.

Skills & Concepts
* read, write, compare, and order integers in mathematical and real world situations

You’ll need
* Three Useful Definitions (page A10.6, run 1 copy on a transparency)
* Elevation Bingo (page A10.7, run 1 copy on a transparency, see Advance Preparation)
* Elevation Bingo Board A (page A10.8, run a half class set on colored copy paper)
* Elevation Bingo Board B (page A10.9, run a half class set on white copy paper)
* Word Resource Cards (pages A10.10–A10.12, optional, run 1 copy of each sheet on paper or cardstock)
* 1½" × 2" sticky notes (see Advance Preparation)
* a piece of paper to mask portions of the overhead
* red colored pencils (class set)
* Student Math Journals

Advance Preparation
Cover each of the 9 elevations on the Elevation Bingo overhead with 1½" × 2" sticky notes.

Instructions for Introducing Integers
1. Ask students to get out their math journals and pencils. Explain that you are going to give them three mathematical definitions today, which they will need to record in their journals. Then show just the top portion of the Useful Definitions overhead.
Set A10 Number & Operations: Integers

Activity 1 Introducing Integers (cont.)

Three Useful Definitions

Counting Numbers are the set of numbers we use to count from 1 to infinity.
\{2, 3, 4, 5, 6, 7, ...

Circle the counting numbers. Cross out the others.
1,000  2  -5  0.75  453  35,040

2. Read the text with the class, and clarify as needed. Have students copy the definition into their journals. Then ask them to identify the numbers in the bottom line that fit the definition, and record just those numbers in their journals. When most students are finished, ask a volunteer to name one of the numbers he or she recorded. If he or she is correct, circle the number on the overhead.

   Teacher  Who would like to name one of the numbers from the bottom row they recorded in their journal? Sasha?

   Sasha  I wrote down 1,000.

   Teacher  Thumbs up if you agree with Sasha that 1,000 is a counting number. How do you know?

   Students  Because you land on 1,000 when you're counting. If you count by 1s, you'll get to 1,000. It's just a regular number.

3. Repeat this process until the students have identified, and you have circled, all of the counting numbers: 1000, 2, 453, and 35,040. Ask students to explain why you need to cross out the others in the row. (The counting numbers do not include fractions, decimals, or negative numbers.)

4. Reveal the next section of the overhead and repeat steps 2 and 3. Do the same with the last section. Then give students a few minutes to respond in their journals to the question at the bottom of the overhead.

   Whole Numbers are the set of counting numbers AND zero.
\{0, 1, 2, 3, 4, 5, 6, ...

Circle the whole numbers. Cross out the others.
47  -16  0  3  235  0.25  10

   Integers are the set of whole numbers and their opposites.
\{-3, -2, -1, 0, 1, 2, 3, ...

Circle the integers. Cross out the others.
567  -48  0  0  50  4 -1  -14,890

One example of integers in daily life is a thermometer that measures temperatures above and below 0. List at least 2 other examples of how people use integers at home, in sports, or on the job.

5. Have students pair-share their responses to the question, and then call on volunteers to share their ideas with the class.
**Students** In golf, you can get a score that’s below par. If it usually takes 3 strokes to get the ball into the hole, and you do it in 2 strokes, you get a score of minus 1.
Penalties in football are like negative numbers.
The thermometer at our house goes all the way up to 120° and all the way down to negative 60°.
There are lots of integers in my big sister’s math book.
With money, you can have some, like 5 dollars, or you can have none, that’s zero. Or if you owe someone money, it’s kind of like a negative number.

6. Let the students know that you are going to spend several days investigating integers together. Today, you are going to play a game that involves integers. Divide the class into 2 teams. Give each of the students on Team 1 a copy of Elevation Bingo Board A, and each of the students on Team 2 a copy of Elevation Bingo Board B. Tell them that they will need a red colored pencil to play the game.

7. Place the Elevation Bingo overhead on display. Give students a few moments to examine the display quietly. Then read the text with the class, and ask students to identify the elevation of the ship and the whale in the illustration.

8. Then explain that there is an elevation recorded under each sticky note. You will let teams take turns telling you which sticky notes to remove. If either or both teams have that elevation on their board, they get to circle it and mark the elevation line to show its position. The first team to mark 3 boxes in a vertical, horizontal, or diagonal row wins.

9. Call on a student from one of the teams to tell you which sticky note to remove from the overhead.

**Maya** Please take off the one in the middle of the middle row.

**Students** Okay, the elevation of Imperial, California is 59 feet below sea level.
How could a place on land be below sea level?
It could be in a valley, or a really low place.

**Teacher** If you have 59 feet below sea level on your board, circle it. Then find negative 59 on the elevation line, mark it, and label it.
Students  Where’s 59 on this line?
We need to mark negative 59, not positive 59.
It goes 0, negative 100, and negative 200. Each of the marks in between must be 50.
I’m going to make a mark just a little bit below the negative 50 mark.
59 feet below sea level is like negative 59, right?

10. Call on a student from the other team to tell you which sticky note to remove, and repeat the process described above. Continue until one of the teams has marked 3 boxes in a horizontal, vertical, or diagonal row.

11. When one of the teams has won, remove all of the sticky notes from the overhead. Discuss the set of elevations with the class. Here are some questions and prompts you might pose:

- Which elevation on the overhead is the lowest? Which is the highest?
- List the elevations in order from lowest to highest in your journal. When you are finished, we will list them together on the board so you can check your work.
- How do you know that 72 feet below sea level is lower than 59 feet below sea level?
- How do you know that 125 feet below sea level is lower than 83 feet above sea level? A second grader might be confused because 125 is greater than 83. How would you explain this to a younger student?)
- What is the difference, in feet, between 8 feet below sea level and 30 feet above sea level? Use one of the elevation lines on your sheet to help determine the answer.
- What is the difference, in feet, between 72 feet below sea level and 83 feet above sea level? Use one of the elevation lines on your sheet to help determine the answer.
Activity 1 Introducing Integers (cont.)

Extensions

- Use pages A10.10–A10.12 to create Word Resource Cards for counting numbers, whole numbers, and integers. Post these cards in a prominent location in the classroom for students’ reference.
- The Los Angeles County Office of Education has a website that introduces integers and operations with integers at http://mathstar.lacoe.edu/lessonlinks/integers/integers_main.html. If you have access to the Internet and the necessary projection equipment, you might consider sharing the first two activities in the Introducing Integers section with your students. Integer Challenge helps students understand integers by linking them to extreme temperatures and elevations around the world. Sets of Numbers reviews the definitions of counting numbers, whole numbers, and integers, and uses effective animations to show how the three sets are related. Students who are especially interested in math and/or science might be interested in exploring these interactive computer activities on their own if you don't have time to pursue them with the class.
Three Useful Definitions

**Counting Numbers** are the set of numbers we use to count from 1 to infinity.

\[ 1, 2, 3, 4, 5, 6, 7... \]

Circle the counting numbers. Cross out the others.

\[ 1,000 \quad 2 \quad -5 \quad 0.75 \quad 453 \quad \frac{1}{2} \quad 35,040 \]

**Whole Numbers** are the set of counting numbers AND zero.

\[ 0, 1, 2, 3, 4, 5, 6, 7... \]

Circle the whole numbers. Cross out the others.

\[ 47 \quad -16 \quad 0 \quad \frac{2}{3} \quad 235 \quad 0.25 \quad \frac{5}{10} \quad 1 \]

**Integers** are the set of whole numbers and their opposites.

\[ ...-3, -2, -1, 0, 1, 2, 3... \]

Circle the integers. Cross out the others.

\[ 567 \quad -48 \quad 0 \quad 0.50 \quad \frac{3}{4} \quad -1 \quad -14,890 \]

One example of integers in daily life is a thermometer that measures temperatures above and below 0. List at least 2 other examples of how people use integers at home, in sports, or on the job.
Elevation Bingo

People use integers to describe elevation. The highest elevation on the mountain in this picture is 200 feet above sea level. What is the elevation of the ship? At what elevation is the whale swimming?

<table>
<thead>
<tr>
<th>Elevation of Desert Shores, CA.*</th>
<th>Highest Elevation in Houston, TX.**</th>
<th>Elevation of Salton City, CA.*</th>
</tr>
</thead>
<tbody>
<tr>
<td>–200 feet</td>
<td>83 feet</td>
<td>–125 feet</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Lowest Elevation in New Orleans, LA.**</th>
<th>Elevation of Imperial, CA.*</th>
<th>Lowest Elevation in Memphis, TN**</th>
</tr>
</thead>
<tbody>
<tr>
<td>–8 feet</td>
<td>–59 feet</td>
<td>195 feet</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Elevation of Coachella, CA.*</th>
<th>Lowest Elevation in Portland, OR**</th>
<th>Highest Elevation in Miami, FL**</th>
</tr>
</thead>
<tbody>
<tr>
<td>–72 feet</td>
<td>Sea Level</td>
<td>30 feet</td>
</tr>
</tbody>
</table>

* Wikipedia@http://en.wikipedia.org/wiki/List_of_places_on_land_with_elevations_below_sea_level
## Elevations Bingo

### Board A

<table>
<thead>
<tr>
<th>200 feet below sea level</th>
<th>59 feet below sea level</th>
<th>30 feet above sea level</th>
</tr>
</thead>
<tbody>
<tr>
<td>200'</td>
<td>0'</td>
<td>200'</td>
</tr>
<tr>
<td>100'</td>
<td>-100'</td>
<td>100'</td>
</tr>
<tr>
<td>0'</td>
<td>-200'</td>
<td>0'</td>
</tr>
<tr>
<td>-100'</td>
<td>-200'</td>
<td>-100'</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>125 feet below sea level</th>
<th>72 feet below sea level</th>
<th>At sea level</th>
</tr>
</thead>
<tbody>
<tr>
<td>200'</td>
<td>0'</td>
<td>200'</td>
</tr>
<tr>
<td>100'</td>
<td>-100'</td>
<td>100'</td>
</tr>
<tr>
<td>0'</td>
<td>-200'</td>
<td>0'</td>
</tr>
<tr>
<td>-100'</td>
<td>-200'</td>
<td>-100'</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>83 feet above sea level</th>
<th>8 feet below sea level</th>
<th>195 feet above sea level</th>
</tr>
</thead>
<tbody>
<tr>
<td>200'</td>
<td>0'</td>
<td>200'</td>
</tr>
<tr>
<td>100'</td>
<td>-100'</td>
<td>100'</td>
</tr>
<tr>
<td>0'</td>
<td>-200'</td>
<td>0'</td>
</tr>
<tr>
<td>-100'</td>
<td>-200'</td>
<td>-100'</td>
</tr>
</tbody>
</table>
### Elevations Bingo  Board B

<table>
<thead>
<tr>
<th>Elevations</th>
<th>At sea level</th>
<th>195 feet above sea level</th>
<th>8 feet below sea level</th>
<th>83 feet above sea level</th>
<th>125 feet below sea level</th>
<th>200 feet below sea level</th>
<th>59 feet below sea level</th>
<th>30 feet above sea level</th>
</tr>
</thead>
<tbody>
<tr>
<td>72 feet below sea level</td>
<td>200'</td>
<td>200'</td>
<td>200'</td>
<td>200'</td>
<td>200'</td>
<td>200'</td>
<td>200'</td>
<td>200'</td>
</tr>
<tr>
<td></td>
<td>100'</td>
<td>100'</td>
<td>100'</td>
<td>100'</td>
<td>100'</td>
<td>100'</td>
<td>100'</td>
<td>100'</td>
</tr>
<tr>
<td></td>
<td>0'</td>
<td>0'</td>
<td>0'</td>
<td>0'</td>
<td>0'</td>
<td>0'</td>
<td>0'</td>
<td>0'</td>
</tr>
<tr>
<td></td>
<td>–100'</td>
<td>–100'</td>
<td>–100'</td>
<td>–100'</td>
<td>–100'</td>
<td>–100'</td>
<td>–100'</td>
<td>–100'</td>
</tr>
</tbody>
</table>
Counting Numbers

1, 2, 3, 4, 5, 6...

Counting Numbers:

also called natural numbers, this is the set of numbers used to count from 1 to infinity
Whole Numbers:
1, 2, 3, 4, 5, 6...

Whole Numbers:
the set of counting numbers AND zero
Integers: \(-3, -2, -1, 0, 1, 2, 3, \ldots\)

the set of whole numbers and their opposites
Negative & Positive Temperature

Water freezes at 32 degrees Fahrenheit, but temperatures on Earth can get much colder than that. Some places even report temperatures below 0 in the winter time.

1 This chart shows the average low temperatures for January in several different cities. Mark each one on the thermometer at right. The first one has been marked for you.

<table>
<thead>
<tr>
<th>City or Town</th>
<th>Average Low, January*</th>
</tr>
</thead>
<tbody>
<tr>
<td>Orlando, FL.</td>
<td>48º F</td>
</tr>
<tr>
<td>Tok, AK.</td>
<td>–27º F</td>
</tr>
<tr>
<td>Memphis, TN</td>
<td>30º F</td>
</tr>
<tr>
<td>Nome, AK</td>
<td>0º F</td>
</tr>
<tr>
<td>Little Falls, MN</td>
<td>–2º F</td>
</tr>
<tr>
<td>Honolulu, HI</td>
<td>65º F</td>
</tr>
<tr>
<td>Hoyt Lakes, MN</td>
<td>–11º F</td>
</tr>
</tbody>
</table>

2 Write less than (<) or greater than (>) in each circle to show how some of the temperatures from the chart compare. Use the thermometer to help.

a  48º F   65º F
b  –2º F  –11º F
c  0º F   –27º F
d  –27º F – 11º F

3 Write the temperatures in order from lowest to highest on the lines below.

________   _______   _______   _______   _______   _______   _______  

lowest       highest

* Temperatures listed above are found on the US Weather website@ http://countrystudies.us/united-states/weather/
Set A10 ★ Activity 2

ACTIVITY

Integer Tug O’ War

Overview
Among the real world situations in which negative numbers appear are games such as Jeopardy and Hearts, where players score both positive and negative points, and football, where teams move in positive and negative directions with respect to their own goal line. Integer Tug O’ War is a board game that bears a slight resemblance to football, in that two teams race to be the first to their own goal line, rolling positive and negative numbers to determine their moves. The teacher introduces Integer Tug O’ War to the whole class, and then students play the game in pairs.

Skills & Concepts
★ read, write, compare, and order integers in mathematical and real world situations

You’ll need
★ Introducing Integer Tug O’ War (page A10.18, run 1 copy on a transparency)
★ Integer Tug O’ War Team 1 Game Board (page A10.19, half class set on colored copy paper)
★ Integer Tug O’ War Team 2 Game Board (page A10.20, half class set on white copy paper)
★ dice numbered 1–6 and 4–9 (1 of each per student pair)
★ positive and negative dice (1 per student pair, see Advance Preparation)
★ black and red linear pieces (4 of each color per student pair)
★ transparent spinner overlays (optional, half class set, see Advance Preparation)
★ Student Math Journals

Advance Preparation If you have blank dice or wood cubes, make a half class set of positive and negative dice by marking 3 sides of each die with a plus sign and 3 sides with a minus sign. Although dice are much better than spinners for this game, you can use page A10.21 to make positive and negative spinners if you don’t have access to blank dice or cubes. (If you make spinners instead of dice, each pair of students will need a transparent spinner overlay.)

Note We strongly recommend that you read the rules and play Integer Tug O’ War by yourself or with a partner before you teach this lesson. While the rules may seem complex at first glance, the game is simple but rich in strategic possibilities.

Background for the Teacher: Helping Students Understand Integers In Elementary and Middle School Mathematics: Teaching Developmentally, John Van de Walle writes that the number line is one of the two models most commonly used for teaching integers in middle school. He says that the number line can be confusing, and that games such as football can provide students with an intuitive sense of how the model
works. He writes, “It is important to remember that signed values are directed distances and not points on a line.” Integer Tug O’ War is designed to provide early experiences with this concept. For example, positive and negative 5 can be spotted on the number line in this game, but students also come to understand that integers are actually measured distances from 0. If your marker is sitting on negative 2 and you roll positive 5, you will move 5 spaces to the right, landing on positive 3. If your marker is on positive 3 and you roll negative 6, you will move 6 spaces to the left, landing on negative 3. While not intended to teach operations with integers, Integer Tug O’ War helps students develop the understandings they will need to do in middle school.

**Instructions for Integer Tug O’ War**

1. Place the Introducing Tug O’ War overhead on display. Ask students to examine it quietly for a few moments, and then have them pair-share observations. After a minute or so, call on volunteers to share their observations with the class.

   **Students**  It’s a board for a game with 2 teams.  
   Team 1 has stars on the right side; Team 2 has stars on the left.  
   There are positive and negative numbers on the lines.  
   They go up to positive 10 and down to negative 10.  
   The lines are kind of like the elevation lines from yesterday, but they’re sideways instead of up and down.

2. Let students know that this is, a game board. Today, you are going to teach the class a new game called Integer Tug O’ War, that will help them learn more about integers and how they work. Review the definition of integers (the set of whole numbers and their opposites), and note with students that the members of each number pair (–1 and 1, –2 and 2, –3 and 3, etc.) are the same distance from zero in the opposite direction, while zero is neither positive nor negative.

3. Explain that Integer Tug O’ War is a little bit like football, in that each team tries to get their markers to their own goal line. Team 1’s goal lines are set at positive 10; Team 2’s are set at negative 10. At the be-
Activity 2  Integer Tug O’ War (cont.)

At the beginning of the game, each team places their markers at 0. The teams take turns rolling 2 dice and moving the designated number of spaces. One of the dice is numbered; the other is marked with positive and negative signs. A negative roll results in a move to the left; a positive roll results in a move to the right.

4. Ask students to pair up. Number off so that one student in each pair is assigned the number 1, and the other is assigned the number 2. Explain that all the 1s will play for Team 1, and all the 2s will play for Team 2. Then give each pair a Team 1 and a Team 2 game board, while a student helper places a small handful of black and red linear pieces at each table or cluster of desks. Let students know that you are going to play a demonstration game with the whole class that uses only 2 of the tracks for each team. When everyone understands how the game works, they will play with their partners, and use all 4 of the tracks on their boards.

5. Explain that Team 1 will use black linear pieces as game markers, and Team 2 will use red linear pieces. Have students place their markers at zero on the first 2 tracks on their boards while you set up the overhead game board. Then ask a student from each team to roll the 2 dice and report the results. The team with the greater number gets to start first.

6. Once the starting team has been identified, call a representative up from that team to roll the dice and move one of the markers at the overhead, as the members of that team do the same on their game boards. If there is any confusion about which direction to move, explain that rolling a positive number always results in a move to the right, no matter where you are on the line. Likewise, rolling a negative number always results in a move to the left.
Activity 2 Integer Tug O’ War (cont.)

**Jasmine** I rolled negative 5. That means we have to move backwards, kind of like taking 5 away?

**Sergio** That’s good, though. We want to go that way. Our goal line is down on negative 10. We want to roll negative numbers!

7. Now have a representative from the other team come up to roll and move a marker at the overhead as members of that team do so on their own game boards. Then have the two teams take turns rolling and moving until one team gets both the markers to their own goal line. If students become confused about which direction to move, draw a black arrow pointing to the right and a red arrow pointing to the left to help them remember that a positive roll always means a move to the right, and a negative roll always means a move to the left.

Here are some additional rules to introduce as the game proceeds:

- A team can split a roll between its 2 markers. If a team rolls positive 5, for instance, it can move one of its markers 3 spaces to the right, and the other marker 2 spaces to the right.

- The student who is rolling for a team and moving the marker(s) at the overhead on any given turn is in charge of that move for the whole team. (This is important because there will be more than one way to handle the roll in many cases. There may be some discussion, but the student at the overhead gets to make the final decision.)

- If one of a team’s markers lands on the other team’s goal line, the players get to move that marker back to 0. If, for instance, Team 1 has gotten enough negative rolls to have forced one of their markers down to negative 10, they get to move that marker back to 0 before their next turn. (This is the redemption rule, providing a team down on its luck with new hope of winning the game.)
• In order to win, a team has to land on its own goal line exactly. For example, if Team 2 has one of its markers positioned on -9, the players will have to roll a -1 to win. If they roll a -2 or -3, they can split the roll between their two markers. If one of their markers is already at their goal line, they lose a turn and must wait for their next roll.
• Even after a team has gotten one of its markers to its own goal line, that marker is still in play and can still be moved if necessary.
• A team cannot move its markers off the track. If a move is not possible, the players lose that turn and must wait for the next one.

8. Play the game until one team has won. Then ask students to share some thoughts about the game. What are some strategies they might use to win when they play again with their partner?

Students Sometimes it’s good to split up the roll.
Yeah, like if one of your markers is almost to the goal line, and the other one isn’t, you can take part of the roll to get the first marker to the line, and then use the rest for the other marker.
If you’re getting a lot of bad rolls, it’s good to just let one of your markers land on the other team’s goal line. Then you can move it back to 0.
Yeah, the 0 rule is good. It makes you feel like you still have a chance to win.

9. Then give each student pair a signed die and numbered die, and allow them to use the rest of the instructional period to play the game in pairs. Let them know that they can use 2, 3, or even all 4 of the tracks on their board. Put the 4–9 dice in a place that is easily accessible to all the students, and let them know that they can use a 4–9 die in place of the 1–6 die if they want. Furthermore, they can switch back and forth between the 1–6 and 4–9 dice whenever they want during the game, but they cannot use both at once. Circulate to observe and give assistance as needed. Depending on the needs of your students, you may want to pull a small group together to play the game with you. If more than a few of your students run into difficulties, reconvene the group to work through the problem(s) together using the overhead board. Encourage students to play the game more than once if time allows. If they only used 2 tracks the first time, challenge them to use 3 or 4 the next. The more tracks they use, the more options they will have when it comes to dealing with problematic rolls.

Extensions
• Allow students to revisit the game during free time. The more they play Integer Tug O’ War, the more they will be able to refine their strategies. Playing the game multiple times also strengthens students’ understandings of signed numbers as directed distances, providing the foundation they need to add and subtract integers in middle school.
• Ask students to write about their experiences playing Integer Tug O’ War. Some prompts might include:
  o Do you think this is a fair game? Why or why not?
  o Does it make any difference whether you are on the positive or the negative team?
  o Here’s what I like about this game.
  o Here’s what frustrates me about this game.
  o Here’s what I would do to make this a better game.
Introducing Integer Tug O' War

Team 1

-10 -9 -8 -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 9 10

Team 2

-10 -9 -8 -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 9 10
Run a half-class set on colored copy paper.

Integer Tug O’ War

Team 1 Game Board

NAME

DATE

© The Math Learning Center
Integer Tug O’ War Team 2 Game Board
Optional, run a quarter class set. Cut the sheets in half.

Positive/Negative Spinner

Positive/Negative Spinner
Temperature & Elevation Riddles

The number pairs under each line below tell you the $x$ and $y$ coordinates of the letters that will answer these riddles about temperatures and elevations around the world. The first two letters are filled in for you in the first riddle.

1. At 29,035 feet, this mountain has the highest elevation in the world.

$$\begin{align*}
&M \quad T. \\
&\quad (2, -5) \quad (2, -5) \quad (5, -3) \quad (1, 1) \quad (5, -3) \quad (-2, 6) \quad (5, -3) \quad (-5, 8) \quad (-7, 4)
\end{align*}$$

2. The deepest part of the ocean is 35,838 feet below sea level. It is called the

$$\begin{align*}
&(4, -9) \quad (7, 7) \quad (-4, 3) \quad (3, 3) \quad (3, 3) \quad (5, -3) \quad (-5, -5) \quad (-8, -2) \quad (5, -3) \quad (-2, 6) \\
&(8, -6) \quad (5, -3) \quad (5, -3) \quad (-9, -9)
\end{align*}$$

3. The coldest temperature on earth ($-129^\circ$ F) was recorded in

$$\begin{align*}
&(-4, 3) \quad (-5, -5) \quad (-7, 4) \quad (-4, 3) \quad (-2, 6) \quad (4, -9) \quad (-7, 4) \quad (5, 5) \quad (4, -9) \quad (-4, 3)
\end{align*}$$
**Set A10 ★ Activity 3**

**ACTIVITY**

**4-Quadrant Battleship**

**Overview**
This activity provides students with opportunities to read and write ordered pairs of integers as they appear on a 4-quadrant coordinate grid. After they practice locating and recording ordered pairs, the teacher challenges the class to a modified version of Battleship.

**Skills & Concepts**
- read, write, compare, and order integers in mathematical and real world situations
- locate points defined by ordered pairs of integers
- write an ordered pair for a point in a coordinate plane with integer coordinates

**Instructions for 4-Quadrant Battleship**
1. Place the 4-Quadrant Battleship game board overhead on display. Ask students to examine it quietly for a few moments, and then have them pair-share observations. After a minute or so, call on volunteers to share their observations with the class.

**You’ll need**
- 4-Quadrant Battleship game board (page A10.28, run a double-sided class set and 1 copy on a transparency)
- 7 blue game markers
- overhead pens in red and black
- red and blue colored pencils (class set)
Activity 3  4-Quadrant Battleship (cont.)

Students  It’s a grid for Battleship. I’ve played that game before!
It has positive and negative numbers on it.
Those are integers!
It’s kind of like a combination of a number line and a thermometer or an elevation line.
It’s like you have a regular number line, and then one that goes up and down instead of sideways.
We’ve used grids like this before, but they only had positive numbers on them.

2. Tell students that this is a 4-quadrant coordinate grid, and you will use it to play a modified version of Battleship with them later. First, however, you’re going to ask them to practice finding and naming coordinates on the grid. Give students each a copy of the game board.

3. Draw a large red dot at (–3, 5) on the grid. Ask students to identify the location of the dot as precisely as possible.

![4-Quadrant Battleship Grid](image)

Students  It’s on the left-hand side of the number line that’s going up and down.
It’s up 5 and then you have to go over 3 the negative way.
If it was on the other side, it would be at (3, 5) so maybe it’s at (5, –3).

4. Remind students that the horizontal number line is called the x-axis and the vertical number line is called the y-axis, and. Label the axes on the overhead, and write the ordered pair that describes the location of the red dot:

(–3, 5)

Review the fact that any point on the grid can be named with two numbers or coordinates. The first number is the x-coordinate. It tells the distance from the point where the two lines intersect (the origin) along the x-axis. The second number tells the distance from the origin along the y-axis. Because these two numbers are always written in order (first x, then y), they are called an ordered pair. Some students may remember the order by reminding themselves to go over and then up when they identify the location of a point. That still works on a 4-quadrant grid, but if the x-coordinate is negative, one goes over to the left instead of the right, and if the y-coordinate is negative, one goes down instead of up.

5. Erase the red dot, and draw another at (3, 5). Ask students to pair-share the coordinates for this point, and call it out as a group when you give them the signal by raising your hand. Record the ordered pair on the board to confirm their response.
Activity 3 4-Quadrant Battleship (cont.)

6. Repeat step 5 with several other points on the grid, including (3, -5) and (-3, -5). Then erase the overhead and tell students you are going to write an ordered pair on the board for them to locate on their grids. Write (2, -4) on the board, and ask them to mark that point on their grids with a red pencil. After a moment, invite a volunteer to the overhead to mark the point, and explain how he or she knew where to place it.

Marta  I just went over 2, and then down 4 because it's a negative 4, and that's where I made my red dot.

7. Repeat step 6 with several other ordered pairs:

(-4, -9)  (-8, 6)  (6, -10)  (8, 0)  (0, -7)

8. Now explain that you are going to play a modified version of the game Battleship with the class. First, you will turn off the projector and place 7 game markers on your grid. These will be your ships. You will turn the projector on for just a few seconds to give students a quick peek. Then the students will try to sink your ships by identifying their locations correctly. Show students what you mean by placing a blue game marker on the board. If you place it directly over one of the squares on the grid, it will touch 4 points. Ask students to identify the coordinates for each of the 4 points as you write them on the overhead. Explain that when you play the game, they will only have to name one of the 4 coordinates to sink your ship.

9. Erase the overhead and turn off the projector, as students turn their sheets over and get out their red and blue pencils. Place 7 blue game markers at various locations on the board. Position each so that it covers exactly 4 points, and place at least one marker in each quadrant. Turn on the projector for about 10 seconds, just long enough for students to get some sense of how you have positioned the markers, and then turn it off again.

10. Call on a volunteer to guess where one of your ships is by writing an ordered pair on the board that might identify one of the points the ship is touching. Ask the rest of the students to write the ordered pair below the grid on their paper. If one of your ships is touching the point named by the volunteer, tell the students they have made a hit, and have them mark the point on their own grid in red. If none of
Activity 3  4-Quadrant Battleship (cont.)

your ships is touching the point named by the volunteer, tell the students they have missed, and have
them mark the point on their own grid in blue.

**Teacher**  Who would like to take the first shot at sinking one of my ships. Charlie?

**Charlie**  Okay, I think you have a ship at (6, 8).

**Teacher**  Please write those coordinates on the board so the rest of the class can see. Boys and girls,
please copy that ordered pair on your own sheet below the grid. Okay, I’m going to tell you that’s a
hit. How are you going to show that on your grid?

**Students**  We get to put a red dot at (6, 8) on our grids!
It’s red for the explosion that sunk your ship!

11. Repeat step 10 as many times as necessary. In order to prevent the game from becoming tedious,
give the students hints when they name a point that is near one of your ships.

**Teacher**  (5, –4) is a miss, but you are getting very warm. Try going up 1 and over 2 from there
and see what happens.

When students have hit all 7 of your ships, turn on the projector so they can see exactly where you posi-
tioned the ships.

(6, 8)  (5, –4)  (5, 8)  (7, 3)  (3, 10)  (4, 10)
(–2, –2)  (–3, –3)  (–7, –7)  (5, 4)  (7, 4)  (7, 5)
(–7, –6)  (–7, 6)  (7, 6)
Activity 3  4-Quadrant Battleship (cont.)

Extensions

- Reposition your markers and play the game again. Challenge students to sink your ships with fewer guesses this time.
- Invite a student to act as the leader in the game.
- A search for Battleship on the Internet will turn up a variety of results. Some websites, including Math is Fun at http://www.mathsisfun.com/games/battleship.html allow students to play the game with the computer as a partner. While the version on the Math is Fun website of the game doesn't involve a coordinate grid, it does promote spatial reasoning.

INDEPENDENT WORKSHEET

Use Set A10 Independent Worksheets 1–3 (pages A10.29–A10.34) to provide students with more practice reading, writing, and comparing negative numbers in real world and mathematical contexts.
4-Quadrant Battleship

[Blank coordinate grid with values ranging from -10 to 10 on both axes]
Shapes on a 4-Quadrant Grid

1a Plot the following points on the coordinate grid above.

1. (-5, 2)  
2. (-3, 5)  
3. (3, 5)  
4. (5, 2)

5. (5, -2)  
6. (3, -5)  
7. (-3, -5)  
8. (-5, -2)

b Connect the dots in order. Then connect the last dot to the first dot. What is the name of this shape?

2a If you plot the points listed below on the coordinate grid above and connect the dots, what shape will you get?

Points: (-4, 7)  (3, 4)  (-6, -4)

b How do you know for sure?

c Plot the points and connect the dots to find out if you are correct.

(continued on back)
3a Six points have been marked on the coordinate grid above. List the coordinates for each point below. The first one has been done for you.

$$(-5, 0)$$

$$(_, _)$$

$$(_, _)$$

$$(_, _)$$

$$(_, _)$$

$$(_, _)$$

$$(_, _)$$

b Connect the dots. Then connect the last dot to the first dot. What is the name of this shape?

4 Ramani wants to program her robot to walk in a square on this grid. She wants to include all 4 quadrants in the path. List the coordinates for 4 points that would work. Then plot them on the grid above and connect them to check.

$$(_, _)$$

$$(_, _)$$

$$(_, _)$$

$$(_, _)$$

$$(_, _)$$

$$(_, _)$$
Set B1 ★ Activity 1

The Carnival

Overview
Students practice writing equations and drawing diagrams to go with a variety of problem situations. Then they select equations and diagrams that best represent a problem situation.

Skills & Concepts
- represent an unknown quantity using a letter or a symbol
- express mathematical relationships using equations
- use diagrams and equations to draw conclusions about problem situations

You’ll need
- The Carnival (page B1.3, run 1 copy on a transparency)
- More Carnival Problems (pages B1.4–B1.6, run 1 copy on a transparency, plus a class set)
- overhead pens
- piece of paper to mask parts of the overhead
- Student Math Journals or 1 piece of lined or grid paper per student

Instructions for The Carnival
1. Place the top portion of The Carnival overhead on display, keeping the other 3 problems covered for now. Read the problem with the class, and ask students to give the thumbs-up sign when they have the answer. Invite a couple of volunteers to share and explain their solutions.

Gabe  It’s $17.00 because you have to pay $5.00 to get in and $1.00 for each ticket. If you get 12 tickets, that’s $12.00. Five more dollars makes 17 in all.

Alyssa  I said the same thing. I just went 12 + 5 because I knew it was a dollar for every ride ticket.

2. Have students get out their math journals. Ask them to write an equation to show the amount of money it would take to get in and buy 12 ride tickets. Have them pair-share their responses and then call on volunteers to read theirs to the class. Record the suggested equations at the overhead.

3. Explain that sometimes people use a letter, such as \( x \), to represent the quantity to be determined. How would you write the equation if you used \( x \) to stand for the total amount of money in this problem? Discuss this with the class and record their ideas at the overhead.
The Carnival

1. There's a big carnival every year in our town. It's opening tonight. It costs $5.00 to get in and $1 for every ride ticket you buy. How much does it cost to get in and buy 12 ride tickets?

2. Miguel is going with his friend, Corey. Miguel is planning to buy 16 ride tickets. Which equation could be used to find \( x \), the amount of money he'll need to get in and buy ride tickets?
   - a) \( x = 5.00 + (16 \times 1.00) \)
   - b) \( x = 5.00 \times (16 \times 1.00) \)
   - c) \( x = 16.00 - 5.00 \)
   - d) \( x = (16 \times 1.00) ÷ 5.00 \)

3. Corey has $27.00 to spend on admission and tickets. How many tickets will he be able to buy? Write an equation in your journal to show. Use the letter \( x \) to stand for the number of tickets Corey will be able to buy.

4. Miguel's favorite ride is the Teacup. Some of the cups hold 2 people. Others hold 4 people. There are 9 cups in all, and when the ride is full, it holds 24 people. Which diagram best represents this problem? Why?
   - a)
   - b)
   - c)
   - d)

5. Ask students to use the information in problem 4, including diagram c to solve the following problem: How many of the teacups seat 2 people, and how many seat 4 people?

6. Give students each a copy of More Carnival Problems. Review the sheets with the class. When students understand what to do, have them go to work. Encourage them to share and compare strategies and solutions as they work.

7. Reconvene the class as time allows to discuss solutions and strategies for some or all of the problems.

INDEPENDENT WORKSHEET

See Set B1 Independent Worksheets 1 and 2 for more practice selecting equations and diagrams to represent meaningful problem situations.
The Carnival

1 There’s a big carnival every year in our town. It’s opening tonight. It costs $5.00 to get in and $1 for every ride ticket you buy. How much does it cost to get in and buy 12 ride tickets?

2 Miguel is going with his friend, Corey. Miguel is planning to buy 16 ride tickets. Which equation could be used to find \( x \), the amount of money he’ll need to get in and buy ride tickets?

- \( x = 5.00 + (16 \times 1.00) \)
- \( x = 5.00 \times (16 \times 1.00) \)
- \( x = 16.00 - 5.00 \)
- \( x = (16 \times 1.00) \div 5.00 \)

3 Corey has $27.00 to spend on admission and tickets. How many tickets will he be able to buy? Write an equation in your journal to show. Use the letter \( x \) to stand for the number of tickets Corey will be able to buy.

4 Miguel’s favorite ride is the Teacup. Some of the cups hold 2 people. Others hold 4 people. There are 9 cups in all, and when the ride is full, it holds 24 people. Which diagram best represents this problem? Why?
More Carnival Problems  page 1 of 3

1 Each of the seats on the giant ferris wheel holds 3 people. There are 26 seats in all. Which equation could be used to find $x$, the number of people riding when the ferris wheel is full?

- $3 \div 26 = x$
- $x + 26 = 3$
- $3 \times 26 = x$
- $x - 3 = 26$

2 After they rode on the Teacup, Miguel and Corey wanted to go on the Yoyo. In order to get there, they had to walk past the Whip. It is three times as far from the Teacup to the Whip as it is from the Whip to the Yoyo. It is 840 yards from the Teacup to the Yoyo. How far is it from the Teacup to the Whip?

a Which diagram below best shows this problem?

b Use the diagram you picked to help solve the problem. Show all of your work.
More Carnival Problems  page 2 of 3

3  Some of the rides take 2 tickets and some of them take 3 tickets.
   a  If Marisa had 17 tickets and used all of them, how many 2-ticket and 3-ticket rides did she take?
      ○  5 two-ticket rides and 4 three-ticket rides
      ○  10 two-ticket rides and 7 three-ticket rides
      ○  3 two-ticket rides and 3 three-ticket rides
      ○  4 two-ticket rides and 3 three-ticket rides

   b  Use numbers, words, and/or labeled sketches to explain your answer to part a.

4  Darius has 9 rides tickets. His sister Deja has 3 more ride tickets than Darius. Their friend Camila has twice as many ride tickets as Deja.
   a  Which equation could be used to find $x$, the number of tickets Camila has?
      $\frac{9 + 3}{2} = x$  $9 \times 3 = x$  $\frac{9 + 3}{2} = x$  $9 \times 3 \div 2 = x$

   b  Use numbers, words, and/or labeled sketches to explain your answer to part a.
More Carnival Problems  page 3 of 3

5  There is a bumper-car ride for little kids next to the hotdog stand. The fence around the ride is a hexagon with 2 long sides that are equal and 4 short sides that are equal.

a  Which diagram below best shows the fence around the bumper car ride?

b  Use the diagram you picked to write and solve an equation for the perimeter of the fence.
Padre’s Pizza

1 It costs $9.50 for a large pizza with cheese at Padre’s Pizza. Each extra topping is $1.00.

a Which equation could be used to find $y$, the amount of money it would cost for a large pizza with 4 extra toppings?

- $y = $9.50 – $4.00
- $y = $9.50 × (4 × $1.00)
- $y = $9.50 + (4 × $1.00)$
- $y = (4 × $1.00) ÷ $9.50$

b Explain your answer to part a. Why did you choose this equation instead of the others?

2 It’s Ty’s birthday. For his party, his mom bought 4 large pizzas with a total of 9 extra toppings.

a Which equation could be used to find $y$, the amount of money she had to pay?

- $y = $9.50 + (9 × $1.00)$
- $y = (4 × $9.50) + (4 × $1.00)$
- $y = $9.50 – (9 × $1.00)$
- $y = (4 × $9.50) + (9 × $1.00)$

b Explain your answer to part a. Why did you choose this equation instead of the others?

(Continued on back.)
3 The marching band went to Padre's after the Friday night football game. They ordered 7 large pizzas with 3 extra toppings each and 4 large pizzas with 4 extra toppings each.

a Which equation could be used to find \( t \), the total number of extra toppings?

- \( t = (7 \times 3) + (4 \times 4) \)
- \( t = 7 \times 3 \times 4 \times 4 \)
- \( t = (7 + 3) \times (4 + 4) \)
- \( t = 7 + 3 + 4 + 4 \)

b Use the equation you picked to solve the problem. How many extra toppings did they order in all? Show your work.

c How much did they have to pay for all the pizzas they ordered? Show all your work.

4 The cook at Padre's Pizza has 12 pizzas lined up for a special order. She put cheese and sausage on all of them. She added pineapple to every second pizza and olives to every third pizza.

a Which pizzas in the line will have all 4 toppings (cheese, sausage, pineapple, and olives)? Mark the row you could use to solve this problem.
Independent Worksheet 1  Padre’s Pizza (cont.)

4b  Which of the 12 pizzas got all 4 toppings?

5  The boy’s basketball team came into Padre’s on Wednesday night after practice. Half the boys on this team also play soccer, \( \frac{1}{4} \) play baseball, and \( \frac{1}{5} \) are in the school band. The remaining 3 boys aren’t in any other activities. No one is in more than 2 activities.

a  How many boys are there on the basketball team? Circle the diagram that will give you the most help solving this problem.

b  Use the diagram you picked to help solve the problem. Show all of your work.
Choosing Equations & Diagrams

Select the diagram and equation that best represent each problem situation below.

1. There are 5 rows of 6 desks in the classroom. Today, 3 of the desks are empty. How many students are in class today?

   a. Which diagram below best shows this problem?

   ![Diagram Options]

   b. If \( x \) represents the number of students in class, which equation could be used to solve the problem?

   ![Equation Options]

   c. Explain your answer to part b. Why did you choose this equation instead of the others?

   (Continued on back.)
2  A pentagon has three longer sides that are all the same length and two shorter sides that are both the same length.

a  Which diagram shows the pentagon described above?

b  Which equation could be used to find the perimeter of the pentagon?

3  Destiny is having a party. She wants to get two cookies for each of the 8 people, including herself, who will be at the party. If each cookie costs 50¢, how much money will she spend on cookies?

a  Which diagram below best shows this problem?

b  Explain your answer to part a. Why did you choose this diagram instead of the others?

(Continued on next page.)
C If \( x \) represents the amount of money Destiny is going to spend, which equation could be used to solve the problem?

\[
(2 + 8) \times 0.50 = x \\
(2 \times 8) \times 1.00 = x \\
(2 \times 8) - 0.50 = x \\
(2 \times 8) \times 0.50 = x
\]

4 There are 4 bikes, 2 skateboards, and a tricycle in Milo's garage. How many wheels are there altogether?

a Which diagram below best shows this problem?

b If \( x \) represents the number of wheels in Milo's garage, which equation could be used to solve the problem?

\[
2 \times (4 + 2 + 1) = x \\
(2 + 4 + 3) \times 2 = x \\
2 \times 4 \times 3 = x \\
(4 \times 2) + (2 \times 4) + 3 = x
\]

C Explain your answer to part b. Why did you choose this equation instead of the others?

5 There are some bikes and trikes on the playground. There are 36 wheels in all, and 15 bikes and trikes. How many bikes are there? How many trikes are there? Make a labeled diagram to solve the problem. Show your work. Use the back of the page if you need more room.
Introducing the Standard Algorithm

Overview
Chances are, many of your students are using the strategies taught in Unit Four with pretty good success by now. There are times, however, when the multiplication menu is not the most efficient or effective method to divide one number by another. This activity introduces the standard algorithm for long division as another method.

Skills & Concepts
- fluently and accurately divide up to a 4-digit number by 1- and 2-digit divisors accurately using the standard long division algorithm
- estimate quotients to approximate solutions and determine reasonableness of answers in problems involving up to 2-digit divisors
- determine and interpret the mean of a small data set of whole numbers

Instructions for Introducing the Standard Algorithm
1. Let students know that you are going to introduce a strategy for long division that may be new to some of them, and familiar to others. Place the top portion of the first overhead on display as students get out their journals and pencils.

2. Read the information on the overhead with the class. Review the definition of the term mean, and ask students to record an estimate in their journals, along with a brief explanation of their thinking. After a minute or two, ask them to pair-share their estimates. Then call on volunteers to share their estimates with the class and explain their thinking.
**Marcus** I said the average is going to be around 280. The first school is almost 300. The second one is more than 300, but the third school is a little less than 250. I think the third school is going to bring the average down to around 280.

**Elisha** I pretty much agree with Marcus, but I think the average is going to be around 275.

3. Now show the bottom portion of the overhead. Review with students how to find the mean by adding and then dividing. Ask them to add the three numbers in their journals, but go no further for now. Have them raise their hands when they have the total. When most hands are raised, call on a few students to share their answers. When there is general consensus that the total is 863, work with student input to record the division problem on the grid that has been provided.

4. Think with students about how using the multiplication menu would play out for this problem. What if you started with $10 \times 3$, then $20 \times 3$, then $5 \times 3$, as you have so many times in solving long division problems this year. Would this information be useful and helpful? Does it seem as if the multiplication menu would be an effective and efficient strategy for solving this problem? Let students pair-share for a minute about these questions.

5. Then explain that there is another strategy that might be easier in this situation. It is called the “standard algorithm” for long division because it is a common paper-and-pencil method for finding a quotient. When people use this strategy, they work with the numbers in the divisor separately. Tell students you are going to demonstrate the strategy. Ask them to watch closely to see if they can understand what you are doing. Challenge them to watch for some of the differences and likenesses between the standard algorithm and the multiplication menu strategy.

**Teacher** First I look at the 8 in 863 and think, “8 divided by 3 is more than 2, just not more than 3, because $2 \times 3$ is 6, and $3 \times 3$ is too much.” So I write a 2 in the hundreds place. Then I write 6 under the 8 and subtract. That’s 2, so I bring down the 6. Now I divide 26 by 3. I get 8 with 2 left over since $8 \times 3$ is 24. So I write an 8 in the tens place and subtract 24 from 26. Does it look like the average is going to be close to your estimate?

**Students** I’ve seen this way to divide from my sister.

It looks like it’s going to come out to two eighty-something.

I think maybe 275 is a little too low.

6. Continue until the problem is complete. Then discuss the remainder with the students. What does a remainder of 2 mean in this context? Would it make best sense to express the remainder as a whole number, a decimal, or a fraction? Why?
Students

You can't cut up the 2. These are kids, not cookies!

If you put exactly the same number of kids in each school, there would be 287 in one school and 288 in the other two.

The average number of students is 287.
The is a remainder of 2. It's kids, so we'll leave the number whole.

7. Ask students to compare the answer with their estimates.
   • Is 287 with a remainder of 2 a reasonable answer?
   • Why or why not?

Invite them to comment on the long division algorithm as well.
   • How does it compare with the multiplication menu?
   • Do they think this strategy would be equally useful in all contexts?
   • Why or why not?

8. After students have had a chance to share their thinking, display the second overhead. Repeat the steps described above, but this time, ask students to work the problem with you in their journals.

<table>
<thead>
<tr>
<th>School</th>
<th>Number of Students</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sarah Goode Elementary</td>
<td>397</td>
</tr>
<tr>
<td>Hayes Elementary</td>
<td>423</td>
</tr>
<tr>
<td>Carver Elementary</td>
<td>229</td>
</tr>
<tr>
<td>Grover Elementary</td>
<td>486</td>
</tr>
</tbody>
</table>

Do you think that the average number of students in the Emerald schools is greater or less than the average number of students in the Jewel Schools? Why?

Estimate the average.

Find the average.
9. Now display the problems on the More Long Division Problems overhead one at a time. Each time, ask students to generate a word problem to match, and record an estimate in their journals, along with a brief explanation of their thinking. You can also ask them apply what they know about divisibility to predict whether or not there will be a remainder. Have them record each problem in their journals, using the grid lines to help align the numbers correctly, and work it as you do so at the overhead. If some of your students are already very familiar with the algorithm, you might let them take turns leading the class at the overhead.

10. Finally, give students each a copy of Using the Standard Algorithm for Long Division. Review the instructions on the first sheet with the class. When students understand what to do, let them go to work. Depending on the strengths and needs of your students, you might give them the choice of working on the sheet independently or working as a smaller group with you.
Schools in Two Towns  page 1 of 2

1 There are 3 elementary schools in Jewel. The chart below shows how many students there are in each school.

<table>
<thead>
<tr>
<th>School</th>
<th>Number of Students</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lincoln Elementary</td>
<td>296</td>
</tr>
<tr>
<td>Washington Elementary</td>
<td>322</td>
</tr>
<tr>
<td>King Elementary</td>
<td>245</td>
</tr>
</tbody>
</table>

a What is the average (mean) number of students in the Jewel elementary schools?

b Estimate the average.

c Find the average.
2. There are 4 elementary schools in Emerald. The chart below shows how many students there are in each school.

<table>
<thead>
<tr>
<th>School</th>
<th>Number of Students</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sarah Goode Elementary</td>
<td>397</td>
</tr>
<tr>
<td>Hayes Elementary</td>
<td>423</td>
</tr>
<tr>
<td>Carver Elementary</td>
<td>229</td>
</tr>
<tr>
<td>Grover Elementary</td>
<td>486</td>
</tr>
</tbody>
</table>

a. Do you think that the average number of students in the Emerald schools is greater or less than the average number of students in the Jewel Schools? Why?

b. Estimate the average.

c. Find the average.
More Long Division Problems

1

\[
\begin{array}{cccc}
5 & 9 & 8 & 5 \\
\end{array}
\]

2

\[
\begin{array}{cccc}
6 & 8 & 0 & 4 \\
\end{array}
\]

3

\[
\begin{array}{cccc}
3 & 8 & 4 & 5 \\
\end{array}
\]

4

\[
\begin{array}{cccc}
9 & 3 & 7 & 4 \\
\end{array}
\]
The standard algorithm is not the only strategy for long division. However, many people find it especially useful when they are dividing a very large number, like 8,746 by a very small number, like 5.

For each of the long division problems on this page and the next:
- write a story problem to match.
- estimate the answer and write a sentence to explain your estimate.
- predict whether there will be a remainder or not, and explain your thinking.

**Example**

```
| 1 | 7 | 4 | 9 | r1 |
+---+---+---+---+---+
|   | 5 | 8 | 7 | 4 |
+---+---+---+---+---+
|   | 3 | 7 |   |   |
+---+---+---+---+---+
|   | 3 | 5 |   |   |
+---+---+---+---+---+
|   | 2 | 4 |   |   |
+---+---+---+---+---+
|   | 2 | 0 |   |   |
+---+---+---+---+---+
|   | 4 | 6 |   |   |
+---+---+---+---+---+
|   | 4 | 5 |   |   |
+---+---+---+---+---+
|   |   |   |   | 1 |
```

**Story Problem**

**Estimate:**

**Remainder or Not?**

**Story Problem**

**Estimate**

**Remainder or Not?**
### Using the Standard Algorithm for Long Division

#### Story Problem

<table>
<thead>
<tr>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>6 8 2 7</td>
<td>5 7 4 2 0</td>
</tr>
</tbody>
</table>

#### Estimate:

#### Remainder or Not?

<table>
<thead>
<tr>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>6 8 2 7</td>
<td>5 7 4 2 0</td>
</tr>
</tbody>
</table>

#### Estimate:

#### Remainder or Not?

<table>
<thead>
<tr>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

© The Math Learning Center
**Set A4 ★ Activity 2**

Extending the Standard Algorithm

**Overview**
Students use the standard algorithm to divide 3- and 4-digit numbers by 2-digit numbers. In the process, they explore the idea of using a “mini” multiplication menu if and when needed.

**Skills & Concepts**
- Fluently and accurately divide up to a 4-digit number by 1- and 2-digit divisors accurately using the standard long division algorithm
- Estimate quotients to approximate solutions and determine reasonableness of answers in problems involving up to 2-digit divisors
- Read and interpret a line plot
- Determine and interpret the mean of a small data set of whole numbers

**You’ll need**
- Family Math Night (page A4.15, run one copy on a transparency)
- Family Math Night Worksheet (pages A4.16 and A4.17, run a class set)
- A piece of paper to mask parts of the overhead
- Overhead pens
- Student Math Journals or 1 piece of lined or grid paper per student

**Instructions for Extending the Standard Algorithm**
1. Open the activity by explaining that the class is going to think some more about strategies for handling long division problems today. Then place the top portion of the Family Math Night overhead on display. Read the text with the class, and give students a minute or two to examine the line plot quietly. Ask them to think of at least two observations they can share with a partner in a minute.
Activity 2  Extending the Standard Algorithm (cont.)

2. Have students pair-share their observations about the line plot. Then pose the following questions about the line plot.
   - What information does the line plot provide?
   - Who might be interested in this information?
   - What does each x stand for? (a classroom)
   - How many classrooms brought 14 families to Math Night? (8 classrooms)
   - How many families was that in all? (112 families)
   - About how many families in all came to Math Night?
   - Can you find the exact total by counting up the x's? Why not?
   - What do you need to do to find the total number of families that came to Math Night?

Be sure students understand that they can’t find the total number of families by simply counting the x’s on the line plot, because each x stands for a classroom. There are 6 x’s above the 15, which means that 6 classrooms brought in 15 families each. $6 \times 15$ is 90, and that’s only part of the total.

3. Now give students each a copy of the Family Math Night Worksheets, and display the prompt toward the bottom of the Family Math Night overhead that instructs students to use the information on the line plot to answer the questions on their sheets. Be sure students understand they need to stop after question 4b on the second sheet.

4. Read the information on the overhead to the class, and review the worksheets with them. When they understand what to do, let them go to work. Encourage them to share and compare their answers to problems 3, 4a, and 4b, and use scratch paper to make or check their calculations. If their solutions don’t match, challenge them to work together until they can come to consensus.
Activity 2  Extending the Standard Algorithm (cont.)

5. Circulate to provide support as students are working. Ask students who finish early to double-check their answers to 3, 4a, and 4b, and then find a quiet task to do until their classmates have completed their sheets through question 4b.

6. When most students have completed their sheets through 4b, reconvene the class. Confirm with the group that the total number of families is 395, and ask students to explain what they will need to do to find the average number of families per classroom. Set up the division problem on the overhead grid while students do so on their worksheets. Then ask them to set their sheets aside for a few minutes and work the problem as a class while you record at the overhead.

Teacher  We’re going to use the long division algorithm we learned during the last activity, so I’m going to look at the numbers in the dividend one by one. How many times will 28 go into 3?

Students  It doesn’t.
None at all!
You can’t divide 3 by 28, but you can divide 39 by 28.
Yep, 28 goes into 39 one time. Two times 28 is 56, so 2 is way too much.

Teacher  Okay, so I’ll write a 1 above the 9 to show that we’ve divided 39 by 28. Then I’ll subtract 28 from 395. Uh oh, I think I’m in trouble now. I got 115 when I brought down the 5. I don’t have any idea how many times 28 goes into 115.

7. When 115 remains, suggest making a mini-menu for 28 so you don’t have to solve the problem by trial and error. Work with input from the students to jot a quick menu to the side. We find ten times and five times the divisor to be useful in nearly every situation, and many students will use the information to quickly ascertain that 4 × 28 will bring them closest to 115.
Activity 2  Extending the Standard Algorithm (cont.)

8. When you have finished working the problem at the overhead, ask students to replicate your work on their sheets. What did the average number of families per classroom turn out to be? Were their estimates close? Should the remainder of 3 be left as a whole number, or converted to a fraction or a decimal? Why?

   Students  You can't split up families.
   You have to the leave the remainder whole.
   It's like each classroom brought 14 families, and then 3 of the rooms had 15 if you take the average.

9. Before students complete the rest of the second sheet, erase the grid at the bottom of the overhead. Write 684 ÷ 23 into the grid, ask students to copy the problem into their journals, and work it with you, reviewing each step carefully. Chances are, students will agree that a mini-menu is helpful for this problem as soon as they get to the second step, 224 ÷ 23.

10. Repeat step 9 with several other problems. Here are some possibilities:

   509 ÷ 19  835 ÷ 23  5,604 ÷ 17  6,003 ÷ 24

11. When most students are working comfortably with the algorithm, have them complete their second worksheet, or give them time to do so during a designated seatwork period the following day.

Extensions
- Home Connections 52, 60 and 61 all offer more practice with long division. Ask students to use the long division algorithm to solve the problems on these sheets.
- Encourage students to experiment with the full-blown multiplication menu, the long division algorithm, and the mini-menu strategy you introduced today. When is each strategy most useful? Is the standard algorithm for long division always the most efficient and effective?
- Ask students to solve a small set of 3–4 long division problems twice or three times a week during seatwork periods throughout the rest of the school year.
Family Math Night

Every year, King School holds a big math night in the spring. They invite all the families to come from 6:30 to 8:00 pm for refreshments, math games, and prizes. They also award a big prize to the classroom that brings in the most families.

King School has 28 classrooms. The line plot on your sheet shows how many families came to Math Night this year.

Use the information on the line plot to answer the questions on Family Math Night, sheets 1 and 2. STOP after question 4b.
Every year, King School holds a big math night in the spring. They invite all the families to come from 6:30 to 8:00 pm for refreshments, math games, and prizes. They also award a big prize to the classroom that brings in the most families.

King School has 28 classrooms. The line plot below shows how many families came to Math Night this year.

1. How many families did the winning classroom bring to Math Night? _________
2. How many families did most of the classrooms bring in? _________
3. Use the information from the line plot to complete the chart below.

<table>
<thead>
<tr>
<th>Classrooms</th>
<th>Families</th>
<th>Total Number of Families</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>11</td>
<td>1 × 11 = 11 families</td>
</tr>
<tr>
<td>b</td>
<td>12</td>
<td>2 × 12 = 24 families</td>
</tr>
<tr>
<td>c</td>
<td>13</td>
<td>7 × 13 = ___ families</td>
</tr>
<tr>
<td>d</td>
<td>14</td>
<td>___ × 14 = ___ families</td>
</tr>
<tr>
<td>e</td>
<td>___</td>
<td>6 × ___ = ___ families</td>
</tr>
<tr>
<td>f</td>
<td>16</td>
<td>___ × 16 = ___ families</td>
</tr>
<tr>
<td>g</td>
<td>17</td>
<td>___ × ___ = ___ families</td>
</tr>
<tr>
<td>h</td>
<td>18</td>
<td>___ × ___ = ___ families</td>
</tr>
<tr>
<td>i</td>
<td></td>
<td>Grand Total: How many families in all attended Math Night? ___ families</td>
</tr>
</tbody>
</table>
Family Math Night Worksheets  page 2 of 2

4a  Sara thinks the average number of families that came to Math Night from each classroom is 14. Do you agree with Sara? (Circle your answer.)

Yes  No

b  If you think the average is 14 families per classroom, explain why. If you think the average is not 14 families, tell what you think it is and explain why.

c  Use the standard algorithm for long division to find the average number of families per classroom that came to Math Night at King School.

The average number of families per classroom was ______________

5  Copy each of the problems below into your Math Journal. Use the standard algorithm for long division to solve each. Make mini-menus when you need them.

a  

2 6 9 7 8  

b  

2 4 7 8 4  

c  

1 7 6 3 1  

d  

3 2 7 9 6  

e  

4 3 9 9 2  

f  

1 4 5 0 7 7  

g  

2 3 8 5 9 4  

h  

2 6 7 4 0 2  

i  

3 8 9 4 1 0
GRADE 5 – UNIT 8

CCSS SUPPLEMENT ACTIVITIES & INDEPENDENT WORKSHEETS

No Supplements Used
Bridges Grade 5 Correlations to Common Core State Standards

Common Core State Standards for Mathematics, Grade 5

In Grade 5, instructional time should focus on three critical areas: (1) developing fluency with addition and subtraction of fractions, and developing understanding of the multiplication of fractions and of division of fractions in limited cases (unit fractions divided by whole numbers and whole numbers divided by unit fractions); (2) extending division to 2-digit divisors, integrating decimal fractions into the place value system and developing understanding of operations with decimals to hundredths, and developing fluency with whole number and decimal operations; and (3) developing understanding of volume.

(1) Students apply their understanding of fractions and fraction models to represent the addition and subtraction of fractions with unlike denominators as equivalent calculations with like denominators. They develop fluency in calculating sums and differences of fractions, and make reasonable estimates of them. Students also use the meaning of fractions, of multiplication and division, and the relationship between multiplication and division to understand and explain why the procedures for multiplying and dividing fractions make sense. (Note: this is limited to the case of dividing unit fractions by whole numbers and whole numbers by unit fractions.)

(2) Students develop understanding of why division procedures work based on the meaning of base-ten numerals and properties of operations. They finalize fluency with multi-digit addition, subtraction, multiplication, and division. They apply their understandings of models for decimals, decimal notation, and properties of operations to add and subtract decimals to hundredths. They develop fluency in these computations, and make reasonable estimates of their results. Students use the relationship between decimals and fractions, as well as the relationship between finite decimals and whole numbers (i.e., a finite decimal multiplied by an appropriate power of 10 is a whole number), to understand and explain why the procedures for multiplying and dividing finite decimals make sense. They compute products and quotients of decimals to hundredths efficiently and accurately.

(3) Students recognize volume as an attribute of three-dimensional space. They understand that volume can be measured by finding the total number of same-size units of volume required to fill the space without gaps or overlaps. They understand that a 1-unit by 1-unit by 1-unit cube is the standard unit for measuring volume. They select appropriate units, strategies, and tools for solving problems that involve estimating and measuring volume. They decompose three-dimensional shapes and find volumes of right rectangular prisms by viewing them as decomposed into layers of arrays of cubes. They measure necessary attributes of shapes in order to determine volumes to solve real world and mathematical problems.

Taken from the Common Core State Standards for Mathematics 2010, page 33.

Grade 5 Overview

Operations & Algebraic Thinking
- Write and interpret numerical expressions.
- Analyze patterns and relationships.

Number & Operations in Base Ten
- Understand the place value system.
- Perform operations with multi-digit whole numbers and with decimals to hundredths.

Number & Operations—Fractions
- Use equivalent fractions as a strategy to add and subtract fractions.
- Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

Measurement & Data
- Convert like measurement units within a given measurement system.
- Represent and interpret data.
- Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.

Geometry
- Graph points on the coordinate plane to solve real-world and mathematical problems.
- Classify two-dimensional figures into categories based on their properties.

Mathematical Practices
1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.
<table>
<thead>
<tr>
<th>Standard</th>
<th>Bridges</th>
<th>Number Corner</th>
<th>Bridges Supplement</th>
<th>Assessments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Write and interpret numerical expressions.</td>
<td>Unit 1, Sessions 13, 14&lt;br&gt;Work Place 1B&lt;br&gt;Unit 2, Sessions 2, 3, 5–9, 11, 13&lt;br&gt;Unit 4, Session 3&lt;br&gt;Unit 7, Sessions 1, 2&lt;br&gt;Unit 7, p 972 (Challenge)&lt;br&gt;Home Connections 6, 7, 13, 14, 17, 18, 33, 41, 42, 47, 48, 59–61</td>
<td>Sep. Computational Fluency</td>
<td>Bridges Practice Book, pp 11, 12, 121, 122, 124</td>
<td>Formal&lt;br&gt;Bridges, Vol. 1, pp 32–39, 138–144 (Unit 1 Pre- &amp; Post-Assessments)&lt;br&gt;Bridges, Vol. 4, pp 942–947, 1020–1024 (Unit 7 Pre- &amp; Post-Assessments)</td>
</tr>
<tr>
<td>1. Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2. Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. For example, express the calculation “add 8 and 7, then multiply by 2” as $2 \times (8 + 7)$. Recognize that $3 \times (18932 + 921)$ is three times as large as 18932 + 921, without having to calculate the indicated sum or product.</td>
<td>Unit 7, Sessions 4, 5, 7, 9, 11–13&lt;br&gt;Home Connections 52, 61–64</td>
<td>Set B1 Algebra: Diagrams &amp; Equations, Activity 1 &amp; Ind. Worksheets 1, 2&lt;br&gt;Bridges Practice Book, pp 1, 31, 125, 126, 128</td>
<td>Formal&lt;br&gt;Bridges, Vol. 4, pp 942–947, 1020–1024 (Unit 7 Pre- &amp; Post-Assessments)</td>
<td></td>
</tr>
<tr>
<td>3. Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane. For example, given the rule “Add 3” and the starting number 0, and given the rule “Add 6” and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so.</td>
<td>Unit 1, Sessions 5, 6, 16–18&lt;br&gt;Unit 7, Sessions 4–7, 9&lt;br&gt;Home Connections 2, 8, 61–64</td>
<td>Bridges Practice Book, pp 6, 7, 8, 33</td>
<td>Formal&lt;br&gt;Bridges, Vol. 1, pp 32–39, 138–44 (Unit 1 Pre- and Post-Assessments)&lt;br&gt;Bridges, Vol. 4, pp 942–948, 1020–1024 (Unit 7 Pre- and Post-Assessments)</td>
<td></td>
</tr>
</tbody>
</table>

*Note: For online CCSS assessment resources go to: [http://bridges1.mathlearningcenter.org/CCSS](http://bridges1.mathlearningcenter.org/CCSS)
### NUMBER AND OPERATIONS IN BASE TEN 5.NBT

<table>
<thead>
<tr>
<th>Standard</th>
<th>Bridges</th>
<th>Number Corner</th>
<th>Bridges Supplement</th>
<th>Assessments</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Understand the place value system.</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
| 1. Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and \( \frac{1}{10} \) of what it represents in the place to its left. | Unit 2, Sessions 1, 2
Unit 6, Sessions 8–12
Work Place 6C
Home Connections 53, 54, 58 | Nov. Calendar Grid
Feb. Calendar Grid
Mar. Computational Fluency | Set A11 Number & Operations:
Multiplying & Dividing Decimals,
Activities 1, 2 & Ind. Worksheets 1, 2
Bridges Practice Book, pp 111, 112, 113, 130 | Formal
Bridges, Vol 3, pp 779–783, 902–907
(Unit 6 Pre- and Post-Assessments)
Number Corner, Baseline & Checkups 1–4 (* See Gr 5 Revised Number Corner Quarterly Assessments online) |
| 2a. Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10. | Unit 2, Sessions 1, 2
Unit 4, Sessions 3–5
Unit 6, Sessions 8, 11
Home Connection 33 | Nov. Computational Fluency
Dec. Computational Fluency | Set A11 Number & Operations:
Multiplying & Dividing Decimals,
Activities 1, 2, 4
Bridges Practice Book, pp 22, 23 | Formal
Bridges Practice Book, pp 111, 112, 113, 130
Informal
Set A11 Number & Operations:
Multiplying & Dividing Decimals, Ind. Worksheets 5, 8 |
| 3. Read, write, and compare decimals to thousandths. | | | | |
| a. Read and write decimals to thousandths using base-ten numerals, number names, and expanded form, e.g., 347.392 = 3 × 100 + 4 × 10 + 7 × 1 + 3 × (\( \frac{1}{10} \)) + 9 × (\( \frac{1}{100} \)) + 2 × (\( \frac{1}{1000} \)). | Unit 6, Sessions 8–13, 15
Work Place 6C
Home Connections 53, 54, 56, 58 | Nov. Calendar Grid
Feb. Calendar Grid
Bridges Practice Book, pp 111, 112, 113, 130
Formal
Bridges, Vol 3, pp 779–783, 902–907
(Unit 6 Pre- and Post-Assessments)
Number Corner Baseline & Checkups 2, 3, 4 (* See Gr 5 Revised Number Corner Quarterly Assessments online) |
| b. Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons. | Unit 6, Sessions 9, 13, 16, 17
Work Place 6C
Home Connections 51, 53, 54, 58 | Nov. Calendar Grid
Feb. Calendar Grid | | Formal
Bridges, Vol 3, pp 779–783, 902–907
(Unit 6 Pre- and Post-Assessments)
Number Corner, Checkup 4 (* See Gr 5 Revised Number Corner Quarterly Assessments online) |

*Note: For online CCSS assessment resources go to: [http://bridges1.mathlearningcenter.org/CCSS](http://bridges1.mathlearningcenter.org/CCSS)
## Bridges Grade 5 Correlations to Common Core State Standards (cont.)

### NUMBER AND OPERATIONS IN BASE TEN 5.NBT

<table>
<thead>
<tr>
<th>Standard</th>
<th>Bridges</th>
<th>Number Corner</th>
<th>Bridges Supplement</th>
<th>Assessments</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Understand the place value system.</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4. Use place value understanding to round decimals to any place.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Perform operations with multi-digit whole numbers and with decimals to hundredths.</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5. Fluently multiply multi-digit whole numbers using the standard algorithm.</td>
<td>Unit 2, Sessions 10–12</td>
<td>Nov. Computational Fluency</td>
<td>Bridges Practice Book, pp 25, 26, 28, 29, 30, 81</td>
<td>Formal</td>
</tr>
<tr>
<td></td>
<td>Unit 4, Sessions 2, 4–10</td>
<td>Jan. Computational Fluency</td>
<td></td>
<td>Number Corner, Baseline &amp; Checkups 2, 3 (See Gr 5 Revised Number Corner Quarterly Assessments online)</td>
</tr>
<tr>
<td>6. Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.</td>
<td>Unit 1, Sessions 7, 8, 11, 12</td>
<td>Oct. Calendar Collector</td>
<td>Set A4 Number &amp; Operations: Long Division, Activities 1, 2</td>
<td>Informal</td>
</tr>
<tr>
<td></td>
<td>Unit 4, Sessions 2, 4–10</td>
<td>May Computational Fluency</td>
<td></td>
<td>Number Corner, Baseline &amp; Checkups 1–4 (See Gr 5 Revised Number Corner Quarterly Assessments online)</td>
</tr>
<tr>
<td></td>
<td>Home Connections 2–4, 19, 21, 23, 32, 34–37, 49, 52, 57, 58, 60, 61, 64</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Work Place 4B</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Unit 6, Sessions 2, 10</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Home Connections 2–4, 19, 21, 23, 32, 34–37, 49, 52, 57, 58, 60, 61, 64</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*Note: For online CCSS assessment resources go to: [http://bridges1.mathlearningcenter.org/CCSS](http://bridges1.mathlearningcenter.org/CCSS)
**NUMBER AND OPERATIONS IN BASE TEN 5.NBT**

<table>
<thead>
<tr>
<th>Standard</th>
<th>Bridges</th>
<th>Number Corner</th>
<th>Bridges Supplement</th>
<th>Assessments</th>
</tr>
</thead>
<tbody>
<tr>
<td>7. Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.</td>
<td>Unit 2, Sessions 11, 12</td>
<td>Mar. Computational Fluency</td>
<td>Set A11 Number &amp; Operations: Multiplying &amp; Dividing Decimals, Activities 1–8 &amp; Ind. Worksheets 3–7</td>
<td>Informal Set A11 Number &amp; Operations: Multiplying &amp; Dividing Decimals, Ind. Worksheet 8</td>
</tr>
<tr>
<td></td>
<td>Unit 6, Session 14</td>
<td></td>
<td>Bridges Practice Book, pp 28, 34, 38, 112, 113, 114, 115, 116, 120, 130, 137, 138</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Unit 6, p 873 (Challenge)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Home Connections 53 (Challenge), 55, 56, 58</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Mar. Computational Fluency</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**NUMBER AND OPERATIONS—FRACTIONS 5.NF**

<table>
<thead>
<tr>
<th>Standard</th>
<th>Bridges</th>
<th>Number Corner</th>
<th>Bridges Supplement</th>
<th>Assessments</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. For example, $\frac{3}{8} + \frac{5}{8} = \frac{8}{8} = 1$. (In general, $\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}$.)</td>
<td>Unit 4, Sessions 11–16, 19, 20, 21 Work Place 4E</td>
<td>Nov. Calendar Grid Nov. Calendar Collector Mar. Computational Fluency Apr. Computational Fluency</td>
<td>Set A6 Number &amp; Operations: Fraction Concepts, Activities 1, 2 &amp; Ind. Worksheets 1–3</td>
<td>Informal Set A6 Number &amp; Operations: Fraction Concepts, Activities 1, 2 &amp; Ind. Worksheets 1–3</td>
</tr>
<tr>
<td></td>
<td>Home Connections, 23, 40, 51, 52, 58</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*Note: For online CCSS assessment resources go to: [http://bridges1.mathlearningcenter.org/CCSS](http://bridges1.mathlearningcenter.org/CCSS)*
<table>
<thead>
<tr>
<th>Standard</th>
<th>Bridges</th>
<th>Number Corner</th>
<th>Bridges Supplement</th>
<th>Assessments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Use equivalent fractions as a strategy to add and subtract fractions.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>2.</strong> Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. For example, recognize an incorrect result ( \frac{2}{5} + \frac{1}{2} = \frac{3}{7} ), by observing that ( \frac{3}{7} &lt; \frac{1}{2} ).</td>
<td>Unit 4, Sessions 11–16, 19, 20–22&lt;br&gt;Unit 6, Sessions 5–7, 14&lt;br&gt;Unit 6, page 890 (Challenge)&lt;br&gt;Unit 6, page 895 (Challenge)&lt;br&gt;Home Connections 23, 40, 51, 52, 56, 58</td>
<td>Nov. Calendar Grid&lt;br&gt;Nov. Calendar Collector&lt;br&gt;Mar. Computational Fluency&lt;br&gt;Apr. Computational Fluency&lt;br&gt;Number Corner Student Book, p 58</td>
<td>Set A9 Number &amp; Operations: Multiplying Fractions, Activities 1–7 &amp; Ind. Worksheets 1–5&lt;br&gt;Set D2 Measurement: Volume, Activities 4, 5 &amp; Ind. Worksheets 4, 5&lt;br&gt;Bridges Practice Book, pp 78, 80, 93, 94, 119, 133, 134, 135, 136</td>
<td>Informal&lt;br&gt;Work Samples:&lt;br&gt;Unit 4, Sessions 15, 16&lt;br&gt;Unit 6, Sessions 6, 14</td>
</tr>
<tr>
<td>Apply and extend previous understandings of multiplication and division to multiply and divide fractions.</td>
<td>Unit 4, p 567&lt;br&gt;Unit 6, Session 2&lt;br&gt;Unit 6, pp 856–857 (Challenge)&lt;br&gt;Work Place 6D&lt;br&gt;Home Connection 37</td>
<td>Mar. Computational Fluency</td>
<td>Set A9 Number &amp; Operations: Multiplying Fractions, Activities 1–7 &amp; Ind. Worksheets 1–5&lt;br&gt;Set A12 Number &amp; Operations: Dividing Fractions &amp; Whole Numbers Activities 2–7 &amp; Ind. Worksheets 1–3&lt;br&gt;Set D2 Measurement: Volume, Activities 4, 5 &amp; Ind. Worksheets 4, 5</td>
<td>Formal&lt;br&gt;Set A12 Number &amp; Operations: Dividing Fractions &amp; Whole Numbers, Activities 1, 8</td>
</tr>
</tbody>
</table>

*Note: For online CCSS assessment resources go to: [http://bridges1.mathlearningcenter.org/CCSS](http://bridges1.mathlearningcenter.org/CCSS)*
### NUMBER AND OPERATIONS—FRACTIONS 5.NF

<table>
<thead>
<tr>
<th>Standard</th>
<th>Bridges</th>
<th>Number Corner</th>
<th>Bridges Supplement</th>
<th>Assessments</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Apply and extend previous understandings of multiplication and division to multiply and divide fractions.</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4. Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>a. Interpret the product ((\frac{a}{b}) \times q) as (a) parts of a partition of (q) into (b) equal parts, equivalently, as the result of a sequence of operations (a \times b). For example, use a visual fraction model to show ((\frac{2}{3}) \times 4 = \frac{8}{3}), and create a story context for this equation. Do the same with ((\frac{2}{3}) \times (\frac{4}{5}) = \frac{8}{15}). (In general, ((\frac{a}{b}) \times (\frac{c}{d}) = \frac{ac}{bd}).)</td>
<td>Unit 4, Sessions 13, 15, 16</td>
<td>Mar. Computational Fluency Apr. Problem Solving</td>
<td>Set A9 Number &amp; Operations: Multiplying Fractions, Activities 1-7 &amp; Ind. Worksheets 1-5. Set A12 Number &amp; Operations: Dividing Fractions &amp; Whole Numbers Activities 2-7 &amp; Ind. Worksheets 1-3. Set D2 Measurement: Volume, Activities 4, 5 &amp; Ind. Worksheets 4, 5.</td>
<td>Formal</td>
</tr>
<tr>
<td>b. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.</td>
<td>Unit 4, Sessions 13, 15, 16</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5. Interpret multiplication as scaling (resizing) by:</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>a. comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication.</td>
<td>Unit 4, Sessions 3–5, 10</td>
<td>Oct. Calendar Grid</td>
<td>Set A11 Number &amp; Operations: Multiplying &amp; Dividing Decimals, Activity 4</td>
<td>Set A12 Number &amp; Operations: Dividing Fractions &amp; Whole Numbers Activities 2-7 &amp; Ind. Worksheets 1-3.</td>
</tr>
<tr>
<td>b. explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number, and relating the principle of fraction equivalence (\frac{a}{b} = \frac{(n \times a)}{(n \times b)}) to the effect of multiplying (\frac{a}{b}) by 1.</td>
<td>Unit 6, Sessions 3, 4, 5, 6, 7</td>
<td></td>
<td>Set A9 Number &amp; Operations: Multiplying Fractions, Activities 2, 3, 4 Practice Book, pp 103, 104, 106, 127, 129</td>
<td></td>
</tr>
</tbody>
</table>

*Note: For online CCSS assessment resources go to: [http://bridges1.mathlearningcenter.org/CCSS](http://bridges1.mathlearningcenter.org/CCSS)*
### NUMBER AND OPERATIONS—FRACTIONS 5.NF

<table>
<thead>
<tr>
<th>Standard</th>
<th>Bridges</th>
<th>Number Corner</th>
<th>Bridges Supplement</th>
<th>Assessments</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Apply and extend previous understandings of multiplication and division to multiply and divide fractions.</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6. Solve real-world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.</td>
<td>Unit 4, Sessions 13, 15, 16, 20 Unit 4, p 602 Unit 6, Sessions 3, 4, 16 Work Place 68 Home Connections 36, 39, 40, 52, 57, 58</td>
<td>Apr. Problem Solving</td>
<td>Set A9 Number &amp; Operations: Multiplying Fractions, Activities 2–7 &amp; Ind. Worksheets 1–5 Set D2 Measurement: Volume, Activities 4, 5 &amp; Ind. Worksheets 4, 5</td>
<td></td>
</tr>
<tr>
<td>7. Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions. (Note: Division of a fraction by a fraction is not a requirement at this grade.)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>a. Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. For example, create a story context for ( \frac{1}{2} \div 4 ), and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that ( \frac{1}{2} \div 4 = \frac{1}{8} ) because ( \frac{1}{8} \times 4 = \frac{1}{2} ).</td>
<td></td>
<td></td>
<td></td>
<td><strong>Formal</strong> Set A12 Number &amp; Operations: Dividing Fractions &amp; Whole Numbers Activities 2-7 &amp; Ind. Worksheets 1-3</td>
</tr>
<tr>
<td>b. Interpret division of a whole number by a unit fraction, and compute such quotients. For example, create a story context for ( 4 \div \frac{1}{3} ), and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that ( 4 \div \frac{1}{3} = 20 ) because ( 20 \times \frac{1}{3} = 4 ).</td>
<td></td>
<td></td>
<td></td>
<td>Set A12 Number &amp; Operations: Dividing Fractions &amp; Whole Numbers Activities 2-7 &amp; Ind. Worksheets 1-3</td>
</tr>
<tr>
<td>c. Solve real-world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. For example, how much chocolate will each person get if 3 people share ( \frac{1}{2} ) lb of chocolate equally? How many ( \frac{1}{3} )-cup servings are in 2 cups of raisins?</td>
<td>Unit 4, Session 20 Home Connection 40</td>
<td></td>
<td></td>
<td>Set A12 Number &amp; Operations: Dividing Fractions &amp; Whole Numbers Activities 2-7 &amp; Ind. Worksheets 1-3</td>
</tr>
</tbody>
</table>

*Note: For online CCSS assessment resources go to:  [http://bridges1.mathlearningcenter.org/CCSS](http://bridges1.mathlearningcenter.org/CCSS)
Bridges Grade 5 Correlations to Common Core State Standards (cont.)

<table>
<thead>
<tr>
<th>Standard</th>
<th>Bridges</th>
<th>Number Corner</th>
<th>Bridges Supplement</th>
<th>Assessments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Convert like measurement units within a given measurement system.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1. Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real world problems.</td>
<td>Unit 1, Session 1</td>
<td>Nov. Calendar Collector</td>
<td>Set D2 Measurement: Volume, Activities 4, 5 &amp; Ind. Worksheets 4, 5</td>
<td>Formal Number Corner, Checkup 2 (* See Gr 5 Revised Number Corner Quarterly Assessments online)</td>
</tr>
<tr>
<td></td>
<td>Unit 2, Sessions 1, 2</td>
<td>Mar. Calendar Grid</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Unit 4, Sessions 6, 9</td>
<td>Mar. Calendar Collector</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Work Place 4C</td>
<td>Apr. Calendar Collector</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Unit 6, Session 13</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Unit 7, Session 8</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Home Connections 11, 15, 18, 62</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Represent and interpret data.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2. Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Use operations on fractions for this grade to solve problems involving information presented in line plots. For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally.</td>
<td></td>
<td>Set D2 Measurement: Volume, Activities 4, 5 &amp; Ind. Worksheets 4, 5</td>
<td>Informal Set D2 Measurement: Volume, Activity 6</td>
<td></td>
</tr>
<tr>
<td>Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Apr. Calendar Grid</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>a. A cube with side length 1 unit, called a “unit cube,” is said to have “one cubic unit” of volume, and can be used to measure volume.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>b. A solid figure which can be packed without gaps or overlaps using n unit cubes is said to have a volume of n cubic units.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4. Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Jan. Calendar Grid</td>
<td>Set D2 Measurement: Volume, Activities 1–3 &amp; Ind. Worksheets 1–3</td>
<td>Formal Number Corner, Checkups 2, 4 (* See Gr 5 Revised Number Corner Quarterly Assessments online)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Apr. Calendar Grid</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*Note: For online CCSS assessment resources go to: http://bridges1.mathlearningcenter.org/CCSS
**Bridges Grade 5 Correlations to Common Core State Standards (cont.)**

<table>
<thead>
<tr>
<th>Standard</th>
<th>Bridges</th>
<th>Number Corner</th>
<th>Bridges Supplement</th>
<th>Assessments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5. Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>a. Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication.</td>
<td>Unit 3, Session 20</td>
<td>Jan. Calendar Grid Apr. Calendar Grid</td>
<td>Set D2 Measurement: Volume, Activities 1–5 &amp; Ind. Worksheets 1–5</td>
<td>Informal Set D2 Measurement: Volume, Activity 6</td>
</tr>
<tr>
<td>b. Apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real world and mathematical problems.</td>
<td>Apr. Calendar Grid</td>
<td>Set D2 Measurement: Volume, Activities 1–5 &amp; Ind. Worksheets 1–5 Bridges Practice Book, pp 57, 59, 60, 65, 69</td>
<td>Informal Set D2 Measurement: Volume, Activity 6</td>
<td>Formal Bridges, Vol. 2, pp 463–468 (Unit 3 Post-Assessment) Number Corner, Checkups 2, 4 (* See Gr 5 Revised Number Corner Quarterly Assessments online)</td>
</tr>
<tr>
<td>c. Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems.</td>
<td>Jan. Calendar Grid</td>
<td>Set D2 Measurement: Volume, Activities 3–5 &amp; Ind. Worksheets 3–5</td>
<td>Informal Set D2 Measurement: Volume, Activity 6</td>
<td></td>
</tr>
</tbody>
</table>

*Note: For online CCSS assessment resources go to: [http://bridges1.mathlearningcenter.org/CCSS](http://bridges1.mathlearningcenter.org/CCSS)
### Bridges Grade 5 Correlations to Common Core State Standards (cont.)

#### GEOMETRY 5.G

<table>
<thead>
<tr>
<th>Standard</th>
<th>Bridges</th>
<th>Number Corner</th>
<th>Bridges Supplement</th>
<th>Assessments</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Graph points on the coordinate plane to solve real-world and mathematical problems.</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
| 1. Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., x-axis and x-coordinate, y-axis and y-coordinate). | Unit 1, Session 18  
Unit 7, Sessions 4–6  
Home Connection 28 | Mar. Calendar Grid | Set A10 Number & Operations: Integers, Activity 3 & Ind. Worksheets 2, 3 | Formal  
Bridges, Vol. 2, pp 356–361, 463–468 (Unit 3 Pre- and Post-Assessments)  
Number Corner, Checkup 3  
(* See Gr 5 Revised Number Corner Quarterly Assessments online) |
| 2. Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation. | Unit 1, Session 18 | Oct. Computational Fluency  
Mar. Calendar Grid | Set A10 Number & Operations: Integers, Activity 3 & Ind. Worksheets 2, 3  
Bridges Practice Book, pp 55, 98 | |

**Classify two-dimensional figures into categories based on their properties.**

<table>
<thead>
<tr>
<th>Standard</th>
<th>Bridges</th>
<th>Number Corner</th>
<th>Bridges Supplement</th>
<th>Assessments</th>
</tr>
</thead>
</table>
| 3. Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles. | Unit 3, Sessions 1, 8, 13, 14  
Work Place 3B  
Home Connection 24 | Sep. Calendar Grid  
Oct. Calendar Grid | Set C1 Geometry: Triangles & Quadrilaterals, Activities 1, 2 & Ind. Worksheets 1–4  
Bridges Practice Book, pp 41, 43, 44, 97, 140 | Informal  
Bridges Practice Book, pp 41, 43, 97 |
| 4. Classify two-dimensional figures in a hierarchy based on properties. | | | Set C1 Geometry: Triangles & Quadrilaterals, Activities 1, 2 & Ind. Worksheets 1–4 | Formal  
Number Corner, Checkup 3  
(* See Gr 5 Revised Number Corner Quarterly Assessments online) |

*Note: For online CCSS assessment resources go to: [http://bridges1.mathlearningcenter.org/CCSS](http://bridges1.mathlearningcenter.org/CCSS)*