Chapter 5  Properties of Triangles

In Chapter 5 you discovered various properties of triangles. First you studied perpendicular bisectors and angle bisectors. Then you learned that the perpendicular bisectors of a triangle are concurrent and the angle bisectors of a triangle are concurrent. You found out that medians are concurrent, and the altitudes are also concurrent. Then you studied the Midsegment Theorem and learned about various triangle inequalities. You finished the chapter by learning about indirect proof.

Turn to the lesson-by-lesson Chapter Review that starts on p. 310 of the textbook.

Lesson 5.1 Perpendiculars and Bisectors

Important words to know are: perpendicular bisector, equidistant from two points, distance from a point to a line, and equidistant from two lines.

The first goal of Lesson 5.1 is to use properties of perpendicular bisectors. In the figure, \( \overline{AB} \) is the angle bisector of \( \angle BAC \) and the perpendicular bisector of \( \overline{BC} \). You know that \( BE = CE \) by the definition of perpendicular bisector. You also know that \( AB = AC \) by the Perpendicular Bisector Theorem. Because \( DP \perp \overline{AP} \) and \( DQ \perp \overline{AQ} \), you know that \( DP = DQ \) by the Angle Bisector Theorem.

The second goal of Lesson 5.1 is to use properties of angle bisectors to identify equal distances. Any point on the perpendicular bisector of a segment will be equidistant from both endpoints of the segment.

Now try Exercises 1 through 3. If you need help, go to the worked-out Examples on pages 265 through 267.

Lesson 5.2 Bisectors of a Triangle

Important words to know are: perpendicular bisector of a triangle, concurrent lines, point of concurrency, circumcenter of a triangle, angle bisector of a triangle, and incenter of a triangle.

The first goal of Lesson 5.2 is to use properties of perpendicular bisectors of a triangle. The perpendicular bisectors of a triangle intersect at a point, called the circumcenter, that is equidistant from the vertices of the triangle. So for \( \triangle RST \) in Exercise 4, the intersection point \( K \) is equidistant from all three vertices of the triangle.

The second goal of Lesson 5.2 is to use properties of angle bisectors of a triangle. Remember that the word incenter contains the word, in, so it must be inside the triangle.
Chapter Audio Summary for McDougal Littell
Geometry

Now try Exercises 4 and 5. If you need help, go to the worked-out Examples on pages 273 and 274.

Lesson 5.3 Medians and Altitudes of a Triangle

Important words to know are: median of a triangle, centroid of a triangle, altitude of a triangle, and orthocenter of a triangle.

The first goal of Lesson 5.3 is to use properties of medians of a triangle. Note that the medians of \( \triangle ABC \) intersect at \( P \), the centroid of \( \triangle ABC \). Also remember that \( AP = \frac{2}{3} AD \); \( BP = \frac{2}{3} BE \); and \( CP = \frac{2}{3} CF \).

The second goal of Lesson 5.3 is to use properties of altitudes of a triangle. Note the altitudes \( HN \), \( JM \), and \( KL \) of \( \triangle HKJ \). The altitudes of the triangle intersect at \( Q \), the orthocenter of \( \triangle HKJ \).

Now try Exercises 6 through 11. If you need help, go to the worked-out Examples on pages 280 and 281.

Lesson 5.4 Midsegment Theorem

Important words to know are: midsegment of a triangle.

The first goal of Lesson 5.4 is to identify the midsegments of a triangle. A midsegment of a triangle is a segment that contains the midpoints of two sides of the triangle.

The second goal of Lesson 5.4 is to use properties of midsegments of a triangle. In \( \triangle ABC \), \( D \) is the midpoint of \( AC \) and \( E \) is the midpoint of \( BC \). By the Midsegment Theorem, \( DE \) is parallel to \( AB \) and equal to half the length of \( AB \).

Now try Exercises 12 through 15. If you need help, go to the worked-out Examples on pages 287 through 289.

Lesson 5.5 Inequalities in One Triangle

The first goal of Lesson 5.5 is to use triangle measurements to decide which side is longest or which angle is largest. In a triangle, the side and the angle of greatest measurement are always opposite each other. In the diagram, the largest angle, \( \angle MNQ \), is opposite the longest side, MQ. By the Exterior Angle Inequality theorem, the measure of the exterior angle \( \angle MQP \) is greater than the measure of either of the nonadjacent
angles $N$ and $M$. By the Triangle Inequality theorem, the sum of the lengths of any two sides of a triangle is greater than the length of the third side. So $MN + NQ > MQ$, $NQ + MQ > MN$, and $MN + MQ > NQ$.

Now try Exercises 16 through 20. If you need help, go to the worked-out Examples on pages 295 through 297.

Lesson 5.6 Indirect Proof and Inequalities in Two Triangles

Important words to know are: indirect proof.

The first goal of Lesson 5.6 is to read and write an indirect proof. First, identify the statement that you want to prove is true. Begin by assuming that statement is false. Then obtain statements that logically follow from your assumption. If you obtain a contradiction, then the original statement must be true.

The second goal of Lesson 5.6 is to use the Hinge Theorem and its converse to compare side lengths and angle measures. In the Example, $\overline{AB} \cong \overline{DE}$ and $\overline{BC} \cong \overline{EF}$. By the Hinge Theorem, if $m\angle E > m\angle B$, as it is here, then $DF > AC$. By the Converse of the Hinge Theorem, if $DF > AC$, as it is here, then $m\angle E > m\angle B$.

Now try Exercises 21 through 25. If you need help, go to the worked-out Examples on pages 302 through 304.